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Stochastic vibration characteristics of finite element modelled functionally gradient plates

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ABSTRACT

In this paper, stochastic vibration characteristics of finite element modelled functionally gradient plates is investigated. An improved structural kinematics proposed earlier by the authors' which assumes the cubically varying in-plane displacements and quadratically varying transverse displacement across the thickness of the plate is applied. This theory satisfies zero transverse strains conditions at the top and bottom faces of the plate as a priori. The material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power law behaviour in terms of the volume fractions of the constituents. The structural kinematics is implemented with a computationally proficient stochastic C^0 finite element (FEM) based on the first-order perturbation technique (FOPT) to accomplish the second-order response statistics of the graded plates. Convergence and comparison studies have been performed to describe the efficiency of the present formulation, and compared the results with those available in the limited literature. Numerical results have been obtained with different system parameters, temperature rise and boundary conditions.

1. Introduction

A new class of advanced inhomogeneous composite materials distinguished as FGMs are composed of two or more phases with different material properties with continuously varying composition distribution. These materials offer great ability and excellent performance in a wide range of engineering applications. The advantages of using these materials is that they can sustain environments with high temperature gradients, while maintaining their structural integrity. Subsequently, one of the most important applications of FGMs is in the skin panels of supersonic and hypersonic light vehicles, which have to sustain the severe thermo-mechanical loadings [1,2,7].

The manufacturing of these materials are very challenging, because a large number of design variables are involved in the fabrication process. The inadequate control over the manufacturing process leads to the randomness in the structural responses of the FGMs structures. Therefore, deterministic analysis is insufficient to provide complete information about the structural response. Consequently, in the preceding years an effort has been made to develop the stochastic formulations to anticipate the actual vibration characteristics of FGMs plate structure with uncertain system parameters.

Reddy [3] presented theoretical formulation and finite element models by applying third order shear deformation theory for static and dynamic analysis of the FGM plates. Huang and Shen [5] studied nonlinear vibration and dynamic response of FGM plates in thermal environments. The formulations are based on the higher order structural kinematics and general von-Karman type equation, which includes thermal effects. Yang and Shen [6] investigated large deflection and post-buckling responses of FGM rectangular plates by using semi-analytical approach under transverse and in-plane loads. The formulations are based on the Classical plate theory. Sundararajan et al. [4] developed nonlinear formulation in thermal environment based on von-Karman assumptions to study the free vibration characteristics of FGM plates. They obtained nonlinear governing equations using Lagrange's equations of motion and solved using FEM, coupled with direct iterative technique. Kitipornchai et al. [8] presented the random free vibration of functionally graded laminates based on third-order shear deformation theory with general boundary conditions subjected to a temperature change. They adopted the mean-centred first-order perturbation technique to obtain the second-order statistics of vibration frequencies. Yang et al. [9] investigated the effect of randomness on the elastic buckling of

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FGM rectangular plates, resting on an elastic foundation and subjected to uniform in-plane edge compressions. The interaction between the plate and foundation is included in the formulation with a two parameter Pasternak model.

Singh et al. [10] studied the second order statistics of post buckling load of shear deformable laminated composite plates resting on linear elastic foundation with random system properties. The formulation is based on higher order shear deformation plate theory in general von Karman sense. Lal et al. [13] presented the stochastic bending response of laminated composite plates resting on elastic foundation with uncertain system parameters. The transverse shear effects have been included in the system equation using higher order shear deformation theory.

Guo et al. [17] investigated the large-amplitude multi-mode random response of thin shallow shells with rectangular platform at elevated temperatures using a finite element non-linear modal formulation. Salim et al. [28] studied the effect of material parameter randomness on the initial buckling load of rectangular, specially orthotropic, composite laminates based on classical laminate theory. Singh et al. [23] studied the effects of random material properties on buckling of composite plates by using the classical laminate theory and first-order and higher-order shear deformation theories.

Pandit et al. [29] proposed an improved higher-order zigzag theory for vibration of soft core sandwich plates with random material properties. Ibrahim et al. [7] provided a nonlinear finite element model to study the random response of functionally graded material panels subject to combined thermal and random acoustic loads. They derived the governing equations using first-order shear deformable plate theory with von Karman geometric nonlinearity. Jagtap et al. [24] presented the stochastic nonlinear free vibration response of elastically supported functionally graded materials plate resting on two parameter Pasternak foundation having Winkler cubic nonlinearity with random system properties subjected to uniform and nonuniform temperature changes with temperature independent and dependent material properties. Lal et al. [11] examined the effect of sensitivity of randomness in system parameters on the nonlinear transverse central deflection of laminated composite plates subjected to transverse uniform lateral pressure and thermal loading. Lal et al. [14,15] studied the second order statistics of postbukling analysis for FGM plates subjected to mechanical and thermal loadings with square and circular holes.

The determination of an accurate structural behaviour of the graded material primarily depends on the structural kinematics used to model the structure. Various structural kinematics have been extended to improve the relevant study of FGMs plate/shell structures. The classical laminated plate theory may be inappropriate for analysis of the FGM plates, in which volume fractions of two or more materials vary continuously as a function of position in a preferred direction, because it assumes that normal to the mid-plane remains normal during plate deflections. The inaccuracy occurs due to neglecting the effects of transverse shear and normal strains [25]. Due to continuous variation in material properties in a preferred direction the first and higher order shear deformation theory may be conveniently used in the analysis. Recently, several theories have been proposed by the researchers to study the vibration, bending and dynamic response of graded material plates notable among them are, [12,18,21,22,30–40,42].

It is noted that the first-order shear deformation theory proposed by Mindlin [27] does not satisfy the parabolic variation of transverse shear strain in the thickness direction. Consequently, the solution accuracy depends upon the shear correction factor. Generally, in the HSDT kinematics the in-plane displacements are assumed to be a cubic expression of the thickness coordinate and the out-of-plane displacement to be constant. In the present study, the structural kinematics assumes the cubically varying in-plane displacements over the entire thickness, while the transverse displacement varies quadratically to achieve the accountability of normal strain and its derivative in calculation of transverse shear strains. Therefore, the development of an appropriate higher order structural kinematics for describing the thermo-mechanical structural response with uncertain material properties of the advanced functionally graded structures has been of high importance to the researchers in the recent years.

As far as the authors' are aware, large amount of work is available on deterministic analysis of functionally graded and composite plates in thermo-mechanical environment using various structural kinematics, such as, [30,36,35,42,33,40,34,39]. However, no previous work have been reported to accomplish the thermo-mechanical free vibration characteristics of functionally graded material plates with uncertain material properties based on modified structural kinematics [20,21], which assures the cubically varying in-plane displacements over the entire thickness and quadratically varying transverse displacement, to take the effects of normal strain and its derivative in calculation of transverse shear strains. The material properties of the FGM plates are graded continuously in the thickness direction only according to a simple power-law distribution in terms of the volume fractions of the constituent. The governing equations of the vibrated FGM plate are derived using the variational approach. A C^0 continuous finite element based on the first-order perturbation technique with 13 degrees of freedom per node is proposed to minimise the computational exercise required in the disposition of element matrices without compromising the solution credibility. The numerical results have been presented with various system parameters, such as effect of material properties, volume fraction index, vibration characteristics and its dispersion with respect to various random variables, which can be treated as a benchmark for further advanced research.

2. Functionally graded material

A functionally graded material plate made from a mixture of metal and ceramics with length a, width b, and total thickness h is considered in this study. The constituent materials of the plate are varied according to the power-law behaviour from the top to the bottom surface as shown in Fig. 1. The top surface $(z = +h/2)$ of the plate is ceramic rich, whereas the bottom surface $(z = -h/2)$ is metal rich. The formulation are accomplished by considering linear elastic material behaviour with small displacements and strains.

The elastic material properties, like Young's modulus E, Poisson's ratio ν, mass density ρ, thermal expansion coefficient α, of the FGMs plate are treated to be the effective material properties P. These properties are assumed to be vary in the thickness coordinate of the plate according to a function of the volume fractions of the constituents. The effective material properties can be expressed as:

$$P = P_c V_c + P_m V_m = (P_c - P_m)V_c + P_m \tag{1}$$

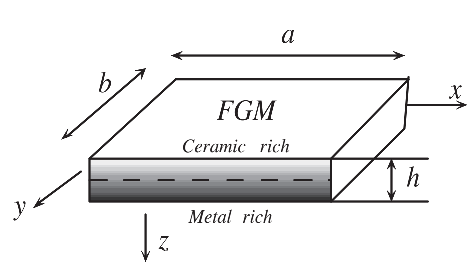

Fig. 1. Geometry and coordinate system of FGMs plate.