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Combined effects of heterogeneity and non-linearity on the critical time parameters of orthotropic conical shells

A.H. Sofiyev [a,*], N. Kuruoglu [b]

[a] Department of Civil Engineering, Faculty of Engineering, Suleyman Demirel University, Isparta, Turkey
[b] Department of Engineering Mathematics, Faculty of Engineering and Natural Sciences, Bahcesehir University, Istanbul, Turkey

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ABSTRACT

The work is devoted to the non-linear dynamic stability of heterogeneous orthotropic truncated conical shells subjected to the combined static and time-dependent axial loads. The basic equations are derived using the finite deflection theory with von Karman–Donnell-type of kinematic non-linearity and reduced to a non-linear differential equation with the time variable coefficient using the superposition principle and Galerkin method. The resulting equation is solved numerically using Runge–Kutta method for variety of an axial loading speed, heterogeneity of features, orthotropic material properties and conical shell characteristics to obtain the non-linear critical time parameters. Finally, the influences of the axial loading speed, non-linearity, heterogeneity and orthotropy on the dimensionless critical time parameters are discussed in detail.

1. Introduction

As a common structure, the composite truncated conical shell has been widely applied in many fields such as space flight, rocketry, aviation, nuclear reactors, jet nozzles, and such other civil, chemical, mechanical, submarine and aerospace engineering technology, etc. The buckling analyses of such shells are very important for their applications and have been of considerable research interest in recent years. The basic information for stability and vibration behaviors of composite shells was described and summarized by Reddy [1]. The fundamental concepts on the present subject—such as geometric non-linearity, bifurcation, and limit loads were interpreted in Refs. [2–4]. The static stability behavior of homogeneous orthotropic conical shells in the small deviation was first studied by Singer [5] and then a number of studies on the stability and vibration behaviors of conical shells have been published, among them are the papers [6–10].

In recent years, new types of composite materials have been used in engineering and many investigations consider heterogeneous orthotropic materials. The heterogeneity of materials stems from the effects of humidity, radiation, surface and thermal polishing processes and methods of production, which render the physical properties of materials, vary from point to point (random, piecewise continuous or continuous functions of coordinates). As heterogeneous materials deform, they retain their shapes up to the point of rupture. Hence, in the computations of structural members made of heterogeneous materials, the fundamental relations and governing equations of deformable body mechanics are applicable [11,12]. In various technological situations are demanding that the heterogeneity of orthotropic materials should be taken into account for the analysis of structural elements. The limited studies available on the behaviors of orthotropic structures with variable material properties [13–16]. Recently, several studies on the linear and non-linear buckling and vibration problems of functionally graded or heterogeneous composite shells have been published in the literature [17–29].

The dynamic stability of shell structures is a complex phenomenon, due to various factors such as inertia effects, non-linearity, loading form, material behaviors, etc. Due to the complexity of this phenomenon, it is not possible to obtain analytical solutions of dynamic stability problems of conical shells under combined static and time-dependent loads in the general case. In the literature, various approaches presented by different researchers (Volmir, Budiansky and Roth, Budiansky and Hutchinson, Simitses; Sachenkov and Bakhtieva) to determine the linear and non-linear critical time parameters [30–34]. So far, there have been several studies on the linear and nonlinear problems of dynamic stability of homogenous cylindrical and conical shells, which have been solved using above mentioned approaches. For instance, Nash and Wilder [35] investigated response of an elastic truncated

* Corresponding author at: Department of Civil Engineering, Faculty of Engineering, Suleyman Demirel University, 32260 Isparta, Turkey. Tel.: +90 246 211 11 95; fax: +90 246 237 08 59.

E-mail address: abdullahavey@sdu.edu.tr (A.H. Sofiyev).

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conical shell to the dynamically applied axial force by means of non-linear equations governing the finite deformations. Shiau et al. [36] studied dynamic buckling of thin truncated conical shells with various geometrical imperfections and under a wide variety of dynamic loading conditions using Galerkin and Runge–Kutta methods. Eslami et al. [37] presented the dynamic buckling and postbuckling of laminated composite cylindrical shells using Layer-wise theory. Shariyat and Eslami [38] investigated the dynamic buckling and postbuckling of imperfect orthotropic cylindrical shells under mechanical and thermal loads, based on the three dimensional theory of elasticity. Lee [39] studied the nonlinear dynamic buckling of orthotropic cylindrical shells subjected to rapidly applied loads using Runge–Kutta method. Sofiyev [40] examined the linear stability of an orthotropic composite truncated conical shell with continuously varying thickness subject to a time dependent external pressure using approach that proposed by Sachenkov and Bakhtieva. Bisagni [41] investigated dynamic buckling due to impulsive loading of thin-walled CFRP cylindrical shells using the finite element code ABAQUS/Explicit. Jabareen and Sheinman [42] presented buckling and sensitivity to imperfection of conical shells under dynamic step-loading. Less and Abramovich [43] studied the dynamic buckling of a laminated composite stringer–stiffened curved panel using the finite element software. Tang and Xu [44] presented nonlinear dynamic buckling for truncated sandwich shallow conical shell structure subjected to the pulse impact. Sina et al. [45] presented experimental and numerical investigation of the stability of composite conical shells subjected to dynamic loading.

The considerable advantages offered by heterogeneous materials over conventional materials and the need of overcoming the technical challenges involving under combined static and dynamic loadings, have prompted an increased use of heterogeneous structural elements. Because of the complexity of derivation and solution of partial differential equations with variable coefficients, only a limited number of studies have been reported for the non-linear dynamic stability of heterogeneous orthotropic shells subjected to combined static and time dependent a-periodic loads. However, most of these investigations are concerned with the dynamic stability analysis of functionally graded (the subgroup of heterogeneous materials) isotropic shells. Shariyat [46] studied dynamic buckling of suddenly loaded imperfect hybrid FGM cylindrical shells with temperature-dependent material properties under thermo-electro-mechanical loads using a modified Budiansky criterion that proposed by the author. Huang and Han [47] presented nonlinear dynamic buckling of functionally graded cylindrical shells subjected to time-dependent axial load using Runge–Kutta method and Budiansky–Roth criterion. Zhang and Li [48] investigated dynamic buckling of FGM truncated conical shells subjected to non-uniform normal impact load. Deniz and Sofiyev [49] and Sofiyev [50] examined the nonlinear dynamic buckling response of FG and FG coated truncated conical shells subject to a time dependent axial load using Runge–Kutta method, respectively. Selahi et al. [51] studied three-dimensional transient analysis of functionally graded truncated conical shells with variable thickness subjected to an asymmetric dynamic pressure.

From the literature survey, one can see that the non-linear dynamic stability of heterogeneous orthotropic truncated conical shell subjected to the combined static and time-dependent axial loads has not been investigated previously. As the geometrical nonlinearity is taken into account in the dynamic stability equations of heterogeneous orthotropic conical shells, unpredictable behaviors may be occur. Therefore, it is very important to develop an accurate, reliable analysis towards the understanding of the non-linear dynamic stability of heterogeneous orthotropic truncated conical shells. The aim of the present paper is to study this problem.

2. Governing equations

As shown in Fig. 1 a thin heterogeneous orthotropic truncated conical shell subjected to combined static and time dependent axial loads is considered. The dynamic part of axial load varying as a linear function of time, i.e., $q(t) = -(q_1 + q_0 t)$, where q_0 is the axial loading speed, q_1 is the static axial load and t is the time. The middle surface of the conical shell is defined as the reference surface. The structure is referred to a curvilinear coordinate system (S, θ, z), where S and θ axes lie along the generator and in the circumferential direction on the reference surface of the cone, respectively and the z axis, being perpendicular to the plane of the first two axes, lies in the inwards normal direction of the cone. R_1 and R_2 indicate the radii of the cone at its small and large ends, respectively, γ denotes the semi-vertex angle of the cone, H is the height of truncated cone, L is the length of truncated cone, h is the thickness of the truncated cone and S_1 is the distance from the vertex to the small base of the conical shell. It is assumed that the local coordinate system, which determines the principal axes of material orthotropy, coincides with the global curvilinear coordinates. The corresponding displacements at the reference are designated u, v and w in the direction of a generator, the circumferential direction, and the inward normal direction, respectively. The stress resultants defined by $N_S = \Psi_{,\theta_1\theta_1}/S^2 + \Psi_{,S}/S$, $N_\theta = \Psi_{,\theta_1\theta_1}/S^2 + \Psi_{,S}/S$ and $N_{S\theta} = -\Psi_{,S\theta_1}/S + \Psi_{,\theta_1}/S^2$, where $\theta_1 = \theta sin\gamma$, Ψ is Airy stress function and a comma denotes partial differentiation with respect to the corresponding coordinates.

The heterogeneity of orthotropic material is assumed to arise due to the variation of Young's moduli, shear modulus and density along the normalized thickness direction $Z = z/h$ as[12,17]

$$[E_S(Z), E_\theta(Z), G(Z), \rho(Z)] = \varphi_1(Z)[E_{0S}, E_{0\theta}, G_0, \rho_0]; \quad Z = z/h \tag{1}$$

where E_{0S} and $E_{0\theta}$ are the Young's moduli in S and θ directions, respectively, G_0 is the shear modulus and ρ_0 is the mass density of a homogeneous orthotropic material. Additionally, $\varphi_1(Z) = 1 + \mu\varphi(Z)$, where $\varphi(Z)$ is the heterogeneity function and defining the variation of Young's moduli, shear modulus and

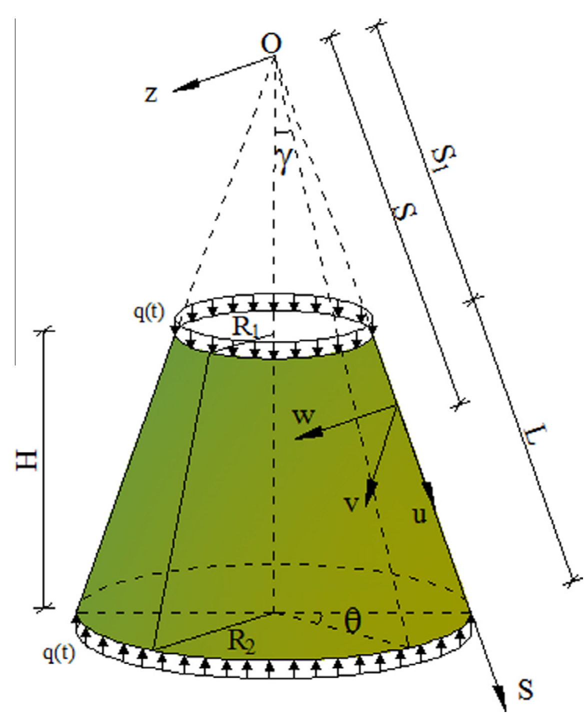

Fig. 1. Geometry of a truncated conical shell subjected to a time dependent axial load.