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A continuum damage model for three-dimensional woven composites and finite element implementation

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ABSTRACT

A continuum damage model for predicting the damage initiation and development in three-dimensional (3D) woven composites is proposed, in which the fiber fracture, inter-fiber fracture and matrix fracture are considered in the level of the fiber yarn and the matrix. A set of damage variables are presented to characterize all the failure modes of the fiber yarn and matrix. The damage initiation and propagation criteria are based on the Puck criteria for the fiber yarn and the paraboloidal yield criterion for the matrix. This continuum damage model is implemented by combining it with the finite element method (FEM). To validate the continuum damage model, the quasi-static tensile experiments of a type of 3D woven composites are carried out, and the tensile failure is predicted by the damage model which agrees well with the experimental results.

1. Introduction

3D woven composites are typical fiber reinforced material which were first developed nearly 40 years ago [1]. Compared with the composite laminates, 3D woven composites can produce complex near-net-shape preforms and have higher delamination resistance and damage resistance.

The failure criteria are the basis of the failure prediction of 3D woven composites. Tsai-Hill criterion [2] was proposed for anisotropic material first and it is the promotion of the von Mises criterion. After that Tsai-Wu criterion [3] and Hoffman criterion [4] were put forward. All of these were classical theories and not able to distinguish the failure modes. Differently, Hashin criterion [5] is formulated under tensile and compressive loads in longitudinal and transverse direction of fiber, respectively. Hashin criterion was improved by Sun et al. [6] and Puck et al. [7,8]. The inter-fiber fiber fracture and inter-fiber fracture are discussed detailedly in the Puck criteria. Davila et al. [9] proposed a failure criteria named by LaRC03 for composites laminates which can predict matrix and fiber failure accurately. Catalanotti et al. [10] proposed a fully 3D failure criteria for polymer composite reinforces which considered the effect of ply thickness.

The continuous damage mechanics (CDM) approach is an important tool for modeling damage evolution in the fiber reinforced composites. Gorbatikh et al. [11] presented a new property degradation procedure to model the damage evolution in textile composites. Bahei-El-Din et al. [12] proposed a damage progression model for multi-scale analysis of 3D woven composites. The RVE of the woven material derived from micrographs is used in his model. Greve and Pickett [13] presented a numerical material model for intra-laminar failure prediction of biaxial non-crimp fabric composites considering effects of fabric pre-shear. Maimí et al. [14,15] proposed a continuum damage model for composite laminates. Fang et al. [16] investigated the compressive properties of the 3D braided composites using the damage theory. Chen et al. [17] proposed a combined elasto-plastic damage model for progressive failure of the fiber-reinforced composites. Bogdanovich et al. [18] presented an experimental study on the damage in the 3D orthogonal woven composites. Cousigné et al. [19] developed a nonlinear numerical material model for textile composite materials considering post-failure damage. Melro et al. [20] developed an elasto-plastic thermodynamically consistent damage model for the matrix in the fiber reinforced composites. Martín-Santos [21] developed a continuum constitutive model for the simulation of fabric-reinforced composites based on CDM.

Most of the above damage models are proposed for 2D composite laminates. As for the limited simulation investigations of the damage evolution behavior in the 3D fiber reinforced composites, some typical limitation can be found in the previously published papers. For example, (1) most of the adopted classical criteria were not able to judge the initial damage precisely and effectively; (2)

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the damage and failure modes were usually not described adequately. Considering the limitation of the above studies, we aim to propose a new continuum damage model for 3D woven composites. In this model, the fiber fracture, inter-fiber fracture and matrix fracture, respectively, can be considered adequately. The failure of a type of 3D woven composites is predicted by this damage model under tension. In order to verify the accuracy of the prediction from this damage model, some typical quasi-static tensile experiments are carried out.

2. Continuum damage model

The material components of 3D woven composites include fiber and matrix. The structure consists of fiber yarns and matrix. The fiber yarn is formed by the fibers and relatively small amounts of permeated matrix. The fiber yarn is assumed to be transversely isotropic. In this section, the constitutive Continuum damage model is proposed for the fiber yarn and matrix of the 3D woven composites. There are three types of failure modes in the 3D woven composites: fiber failure, inter-fiber failure and matrix failure (Fig. 1). It is noted that the failure often occurs in the interface between the fiber yarn and matrix, but there is not interface material and it is the matrix failure, accurately.

2.1. Damage variables and constitutive functions

When the damage initiates and evolves in the material, the mechanical properties are degraded. The damage variables can be used to characterize the damage process. The constitutive function for the fiber yarns can be expressed as:

$$\boldsymbol{\varepsilon}_f = \mathbf{S}_f(d)\boldsymbol{\sigma}_f \tag{1}$$

where $\mathbf{S}_f(d)$ is the compliance matrix, which can be written as:

$$\mathbf{S}_f(d) = \begin{bmatrix} \frac{1}{(1-d_{f,1})E_{f,1}} & -\frac{\nu_{f,12}}{E_{f,1}} & -\frac{\nu_{f,13}}{E_{f,1}} & & & \\ & \frac{1}{(1-d_{f,2})E_{f,2}} & -\frac{\nu_{f,23}}{E_{f,2}} & & 0 & \\ & & \frac{1}{(1-d_{f,3})E_{f,3}} & & & \\ & & & \frac{1}{(1-d_{f,4})G_{f,12}} & & \\ & \text{sym.} & & & \frac{1}{(1-d_{f,5})G_{f,23}} & \\ & & & & & \frac{1}{(1-d_{f,6})G_{f,31}} \end{bmatrix} \tag{2}$$

where the subscript 1 denotes the longitudinal direction of the fiber yarn, the subscript 2 and 3 denote the transverse directions of the fiber yarn, $E_{f,1}, E_{f,2}, E_{f,3}, G_{f,12}, G_{f,23}, G_{f,31}, \nu_{f,12}, \nu_{f,13}$ and $\nu_{f,23}$ are the elastic properties of the fiber yarn, and $d_{f,i}(i = 1 \sim 6)$ is the damage variable. The damage variable $d_{f,1}$ is associated with fiber fracture, and $d_{f,2}$ and $d_{f,3}$ are associated with inter-fiber fracture in the fiber yarn. $d_{f,4}$ and $d_{f,6}$ are associated with fiber fracture and inter-fiber fracture, and $d_{f,5}$ is influenced by inter-fiber fracture. For the convenience of the following derivation, the effective stress $\tilde{\boldsymbol{\sigma}}_f$ can be written as:

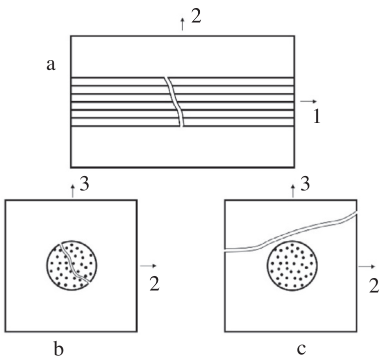

Fig. 1. The failure modes of the 3D woven composites: (a) fiber fracture, (b) inter-fiber fracture and (c) matrix fracture. Axis 1 is along the longitudinal direction of the fiber yarn, axis 2 and 3 are the transverse directions.

$$\tilde{\boldsymbol{\sigma}}_f = \mathbf{S}_{f0}^{-1}(d)\boldsymbol{\varepsilon}_f \tag{3}$$

where $\mathbf{S}_{f0}(d)$ is the undamaged compliance matrix obtained from eq. (2) using $d_{f,i} = 0(i = 1 \sim 6)$.

Similarly, the constitutive function for the matrix can be expressed as:

$$\boldsymbol{\varepsilon}_{\boldsymbol{m}} = \mathbf{S}_{\boldsymbol{m}}(d)\boldsymbol{\sigma}_{\boldsymbol{m}} \tag{4}$$

where $\mathbf{S}_{\boldsymbol{m}}(d)$ is the compliance matrix, which can be written as:

$$\mathbf{S}_{\boldsymbol{m}}(d) = \frac{1}{E_m}\begin{bmatrix} \frac{1}{1-d_m} & -\nu_m & -\nu_m & & & \\ & \frac{1}{1-d_m} & -\nu_m & & 0 & \\ & & \frac{1}{1-d_m} & & & \\ & & & \frac{2(1-\nu_m)}{1-d_m} & & \\ & \text{sym.} & & & \frac{2(1-\nu_m)}{1-d_m} & \\ & & & & & \frac{2(1-\nu_m)}{1-d_m} \end{bmatrix} \tag{5}$$

where E_m, G_m and ν_m are the elastic properties the matrix, and d_m is the damage variable which is associated with matrix fracture. The effective stress $\tilde{\boldsymbol{\sigma}}_m$ can be written as:

$$\tilde{\boldsymbol{\sigma}}_m = \mathbf{S}_{m0}^{-1}(d)\boldsymbol{\varepsilon}_m \tag{6}$$

where $\mathbf{S}_{m0}(d)$ is the undamaged compliance matrix obtained from eq. (5) using $d_m = 0$.

In order to differentiate the effects of the stress state on the failure modes, the damage variables are introduced as follows:

$$d_{f,1} = \begin{cases} d_{f,1t} \text{ if } \tilde{\sigma}_{f,11} \geqslant 0 \\ d_{f,1c} \text{ if } \tilde{\sigma}_{f,11} < 0 \end{cases}, \quad d_{f,i} = \begin{cases} d_{f,it} \text{ if } \tilde{\sigma}_{f,n} \geqslant 0 \\ d_{f,ic} \text{ if } \tilde{\sigma}_{f,n} < 0 \end{cases} (\text{i} = 2, 3),$$
$$d_m = \begin{cases} d_{m,t} \text{ if } I_1 \geqslant 0 \\ d_{m,c} \text{ if } I_1 < 0 \end{cases} \tag{7}$$

where $\tilde{\sigma}_{f,11}$ is the effective stress in the longitudinal direction of the fiber yarn. The effective stress $\tilde{\sigma}_{f,n}$ and the invariant I_1 of stress tensor will be expressed in the following paragraphs.

2.2. Damage initiation and propagation criteria

To predict the damage initiation and propagation in the fiber yarn and matrix and evaluate the mechanical properties, the damage initiation and propagation criteria can be defined as follows:

$$F_{f,N} = \phi_{f,N} - r_{f,N} \leqslant 0, \quad N = \{1t, 1c, 2t, 2c, 3t, 3c\} \tag{8a}$$

$$F_{m,L} = \phi_{m,L} - r_{m,L} \leqslant 0, \quad L = \{t, c\} \tag{8b}$$

where $\phi_{f,N}$ is the loading function for different failure modes for the fiber yarn, $\phi_{m,L}$ for the matrix, and $r_{f,N}$ and $r_{m,L}$ are the damage threshold parameters which are dependent on the damage level and the loading history. The initial values of $r_{f,N}$ and $r_{m,L}$ are 1 and increase when the damage initiates and accumulates. The Puck failure criteria [7,8] is adopted for the fiber yarn as it can predict not only the location but also the orientation of a crack. The loading functions for the fiber yarn can be expressed as:

$$\phi_{f,1t} = \frac{\varepsilon_{f,11}E_{f1} + m_f\nu_{f,12}\tilde{\sigma}_{f,22} + m_f\nu_{f,13}\tilde{\sigma}_{f,33}}{S_{f,1t}} \text{ for } \tilde{\sigma}_{f,11} \geqslant 0 \tag{9a}$$