Composite Structures 128 (2015) 54–69

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Effective elastic properties and stress states of doubly periodic array of inclusions with complex shapes by isogeometric boundary element method

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A R T I C L E I N F O

Article history:
Available online 25 March 2015

Keywords:
Isogeometric boundary element method (IGBEM)
Inclusions
Effective elastic properties
Composite materials

A B S T R A C T

Isogeometric analysis is a new numerical method that has received a lot of attentions in recent years. In this method, the same functions used in the CAD system are used to describe geometry and approximate the field variables in numerical discretization. The isogeometric finite element method (IGFEM) has widely been used to study various problems for a long time, but in the field of the boundary element method (BEM), researches about the isogeometric analysis have not yet attained more attentions. In this paper, we use the isogeometric boundary element method (IGBEM) to carry out the calculation of effective elastic properties and stress states of doubly periodic inclusions with complex shapes. The results obtained by the IGBEM are compared with those from the finite element method (FEM). We believe that this work clearly presents the power of the IGBEM and provides an efficient approach to investigate elastic behavior of composites containing various microstructures.

1. Introduction

In traditional numerical simulation, we first need to build a suitable model for the studied problem and then provide this problem with some boundary conditions. Thus, we can solve the mathematical equations from the obtained model and attain the final results. To today, numerical methods for solving various engineering problems have been popular and mature. So the current challenge works for us are how to create an exact model and reduce the geometric errors when we discretize the models into a series of finite element or boundary element meshes. The "meshing" step for the model, as we have known, will take a lot of time and the final results will also be greatly influenced by different meshes.

The mismatch problems between computer aided design (CAD) and numerical analysis are deserved to be carefully investigated. Hughes et al. [1] proposed the isogeometric concept which have became very popular in the field of computational mechanics. A book about this method has also been published by Hughes and his co-workers [2]. Cottrel et al. [3] studied the vibration of structure by using the IGFEM. Benson et al. [4] used the IGFEM to investigate the Reissner–Mindlin shell. Auricchio et al. [5] adopted isogeometric collocation method to study elastics and explicit dynamics. In the BEM, Simpson et al. [6,7] implemented the IGBEM. But their works are mainly devoted to the 2D elastostatic homogeneous problems, the research about the inhomogeneous problems has not yet been attained more attention. Peake et al. [8] carried out the research about extended isogeometric boundary element method for 2D Helmholtz problems. Li and Qian [9] presented the isogeometric analysis and shape optimization via boundary integral. Since the isogeometric method can allow us do the "no-geometry-error" analysis work and the inhomogeneous problem are sensitive to the shapes of various inhomogeneities, so it is clear that the advantage of the isogeometric method in analyzing the inhomogeneous problems is obvious.

Isogeometric method uses the parametric functions from CAD software to represent geometry and approximate field variables, where the geometry is described using the non-uniform rational B-splines (NURBS), rather than the traditional polynomial shape functions. The best known quality of isogeometric method is that the geometry of the problem is preserved exactly. When building the model using CAD software, we can directly carry out numerical analysis based on the isogeometric method, i.e. there is no transformation from the CAD models to the computational meshes. In addition, significant refinement improvement can be easily obtained using the existing method [1,2].

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http://dx.doi.org/10.1016/j.compstruct.2015.03.061
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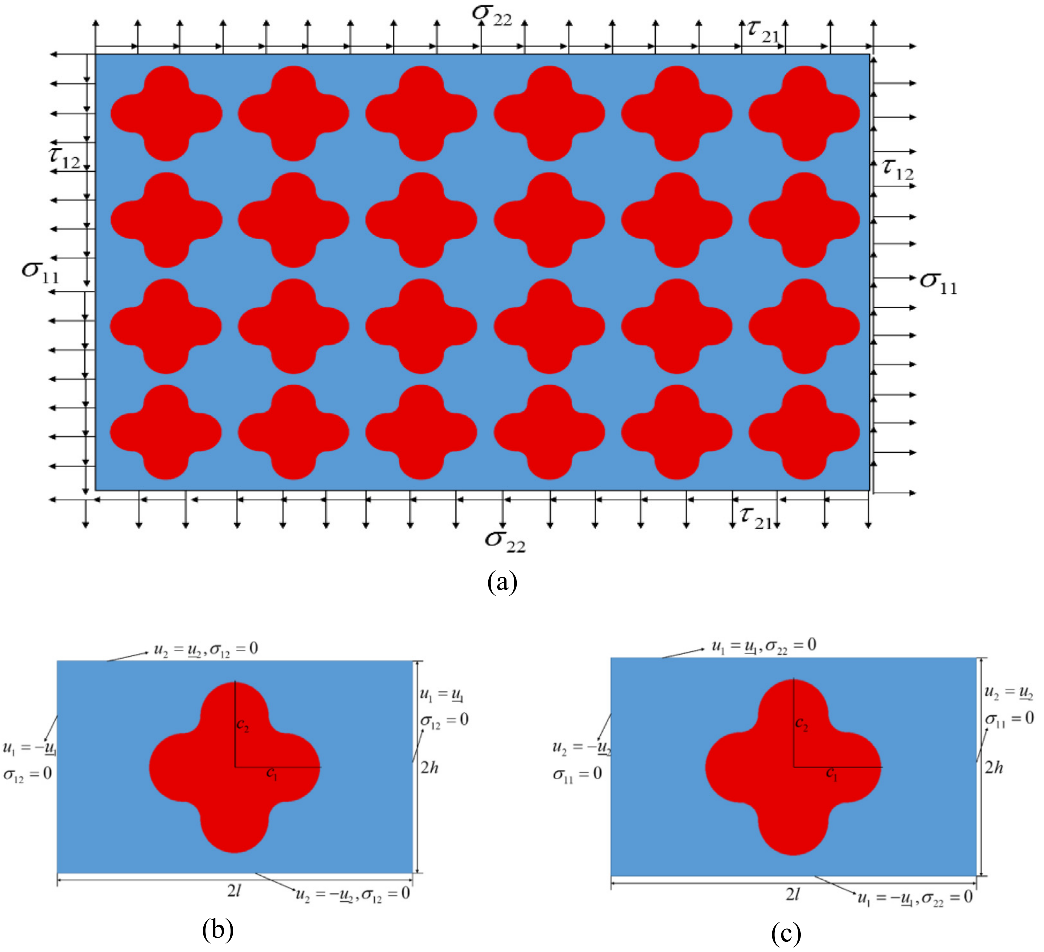

Fig. 1. (a) Doubly periodic inclusions with complex shape; (b) rectangular cell containing single inclusion of complex shape with the boundary conditions which are applied to each edge of rectangular cell for remote tension loading; (c) rectangular cell containing single inclusion of complex shape with the boundary conditions which are applied to each edge of rectangular cell for remote shear loading.

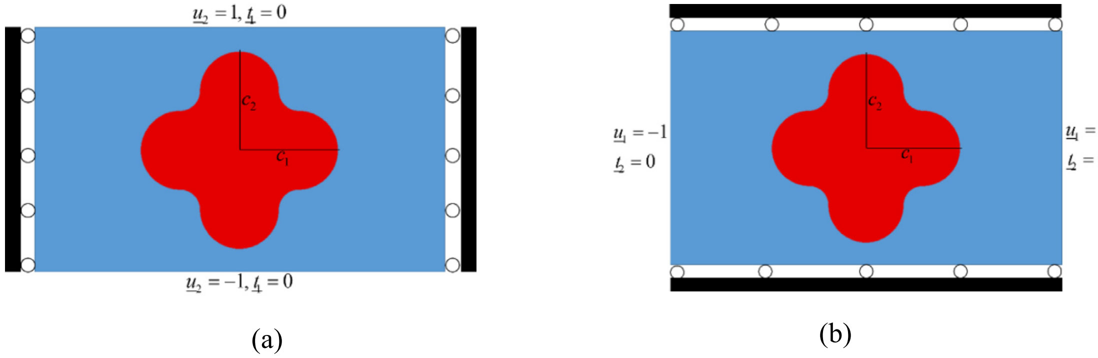

Fig. 2. Sub-problems as shown in Fig. 1(b): (a) $u_1(l, x_2) = u_1(-l, x_2) = 0, u_2(x_1, h) = -u_2(x_1, -h) = 1, \sigma_{12}(-l, x_2) = \sigma_{12}(l, x_2) = 0, \sigma_{12}(x_1, h) = \sigma_{12}(x_1, -h) = 0$; (b) $u_1(l, x_2) = -u_1(-l, x_2) = 1, u_2(x_1, h) = u_2(x_1, -h) = 0, \sigma_{12}(-l, x_2) = \sigma_{12}(l, x_2) = 0, \sigma_{12}(x_1, h) = \sigma_{12}(x_1, -h) = 0$.

In the inhomogeneous problems containing various inclusions with complex shapes, it is very difficult for us to obtain the closed form solutions. Therefore, in order to use analytical or numerical methods to obtain the solution of these problems, one often make some simplifications and give some simple models (e.g. the regular distribution of multiple inclusions). Nemat-Nasser and Hori [10] used the Fourier series expansion method to solve the doubly periodic inclusion problems. The shortage of this method is that the Fourier coefficient of the eigenstrain within periodic inclusions is not easy to be determined. Jiang et al. [11] used the equivalent inclusion method [12] and the results from Lu [13] about the doubly quasi-periodic Riemann boundary value problems to solve the doubly periodic cylindrical inclusion under longitudinal shear. Chen and Lee [14,15] used the eigenfunction expansion variational method [16] to study the doubly periodic circular holes in infinite plane. Dong [17] used an iterative FE–BE coupling method to study the elastostatic problems. Liu et al. [18] adopted the iterative FE–BE coupling method [17] to study the effective elastic properties