Composite Structures 128 (2015) 70–86

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Review

A review of theories for the modeling and analysis of functionally graded plates and shells

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A R T I C L E I N F O

Article history:
Available online 14 March 2015

Keywords:
Functionally graded plate and shell
Plate and shell theory
Elasticity solution
Unified formulation

A B S T R A C T

In this paper, a comprehensive review of various theories for the modeling and analysis of functionally graded plates and shells is presented. The review is devoted to theoretical models which were developed to predict the global responses of functionally graded plates and shells under mechanical and thermal loadings. This review mainly focuses on the equivalent single layer theories including the classical plate theory, first-order shear deformation theory, higher-order shear deformation theories, simplified theories and mixed theories since they were widely used in the modeling of functionally graded plates and shells. In addition, a thorough review of the literature related to the development of three-dimensional elasticity solutions and a unified formulation is also presented.

Contents

1. Introduction

Multilayered composite materials are extensively used in aerospace, mechanics, civil engineering, nuclear and automotive due to their outstanding features such as high ratio of stiffness- and strength-to-weight and low maintenance cost. Conventional laminated composite materials exhibit a mismatch of mechanical properties at an interface due to bonding of two discrete materials. As a result, stress concentration usually occurs at the interface. This can lead to damage in the form of delamination, matrix cracking and adhesive bond separation [1]. Functionally graded materials (FGMs) were therefore born to overcome these issues. The FGM is the advanced composite material which is made of two or more constituent phases with a continuous variation of material properties from one surface to another, and thus eliminating the stress concentration found in the conventional laminated composites. The concept of the FGM was proposed in 1984 by Japanese

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http://dx.doi.org/10.1016/j.compstruct.2015.03.010

material scientists [2]. A typical FGM is made from a mixture of a ceramic and a metal. The history of the FGM as well as its applications can be found in the report by Jha et al. [3]. The modeling and analysis of the FGM were also reviewed by Birman and Byrd [4].

In general, the behavior of functionally graded (FG) plates/shells under mechanical and thermal loadings can be predicted using either three-dimensional (3D) elasticity theory or equivalent-single-layer (ESL) theories. The ESL models are derived from the 3D elasticity theory by making suitable assumptions on the kinematics of deformation or a tress state through the thickness of plates/shells [1]. These ESL theories may account for both shear and normal deformation effects depending on the level of assumptions. The simplest ESL model is the classical plate theory (CPT), also known as Kirchoff theory [5], which ignores both shear and normal deformation effects. Thus it is only suitable for thin FG plates/shells. The next theory in the hierarchy of ESL models is the first-order shear deformation theory (FSDT) developed by Mindlin [6]. The FSDT accounts for the shear deformation effect by the way of a linear variation of in-plane displacements through the thickness. A shear correction factor is therefore required. The shear correction factor is difficult to determine since it depends not only on geometric parameters but also on the loading and boundary conditions. To avoid the use of the shear correction factor, higher-order shear deformation theories (HSDTs) were introduced. The HSDT can be developed by expanding the displacement components in power series of the thickness coordinate. In principle, the theories developed by this mean can be made as accurate as desired by including a sufficient number of terms in the series. Among the HSDTs, the third-order shear deformation theory (TSDT) of Reddy [7] is the most widely used one due to its simplicity and accuracy. A review of shear deformation theories for isotropic and laminated plates was carried out by Ghugal and Shimpi [8] and Khandan et al. [9]. A comprehensive review of various analytical and numerical models for predicting the bending, buckling and vibration responses of FG plates under mechanical and thermal loadings was recently carried out by Swaminathan et al. [10]. However, no literature has been reported for the review of the development of various theories for the modeling and analysis of FG plates/shells.

The objective of this paper is to provide a comprehensive literature review of existing theories for the modeling and analysis of FG plates/shells with the main emphasis on the ESL models such as the CPT, FSDT, TSDT, HSDTs, simplified theories, mixed theories. In addition, a detailed review of the literature related to the development of 3D elasticity solutions and a unified formulation is also reported.

2. ESL theories

2.1. CPT model

The CPT model is based on the Kirchhoff–Love hypothesis that the straight lines remain straight and perpendicular to the mid-plane after deformation. These assumptions imply the vanishing of the shear and normal strains, and consequently, neglecting the shear and normal deformation effects. The CPT is the simplest ESL model and it is only suitable for thin FG plates/shells where the shear and normal deformation effects are inconsiderable.

Feldman and Aboudi [11] studied the elastic buckling of FG plates under uniaxial compressive loading using a combination of micromechanical and structural approaches. Governing equations derived from the CPT were analytically solved for the buckling load of FG plates with various boundary conditions. Javaheri and Eslami [12,13] employed the CPT to investigate the buckling behavior of FG plates under four types of thermal loadings [12] and compressive loadings [13]. Based on the CPT, Kiani et al. [14] presented analytical solutions for the critical buckling temperature of FG clamped plates resting on an elastic foundation under three different types of thermal loadings. Ghannadpour et al. [15] also examined the thermal buckling of FG plates using the CPT. However, the buckling load was calculated using the finite strip method instead of Navier solution in the work [12]. The buckling of FG plates subjected to non-uniform compression was examined by Mahdavian [16] using the CPT and Fourier solutions. Mohammadi et al. [17] derived analytical solutions for the buckling load of FG plates with two opposite edges simply supported and the other two edges having arbitrary boundary conditions (i.e. Levy-type plate). The governing equations derived from the CPT were analytically solved using Levy-type solution approach.

Yang and Shen [18] employed the CPT to investigate the transient response of initially stressed FG plates resting on an elastic foundation subjected to impulsive lateral loadings. The semi-analytical differential quadrature method (DQM) and the modal superposition approach were respectively employed to determine the natural frequency and transient response of rectangular plates with two opposite edges clamped and the remaining edges having arbitrary boundary conditions. The nonlinear load–deflection and postbuckling responses of FG plates resting on an elastic foundation under in-plane and transverse loadings were investigated by Yanga and Shen [19] using the CPT with von Karman assumptions. A semi-analytical approach based on the DQM and Galerkin procedure was used to solve the governing equations. Alinia and Ghannadpour [20] also used the CPT with von-Karman assumptions to study the nonlinear responses of FG plates under transverse pressure. However, they used the principle of minimum potential energy to obtain the analytical solutions of simply supported plates.

Woo et al. [21] studied the nonlinear vibration of FG plates in thermal environments. The nonlinear equations derived from the CPT with von Karman assumptions were solved for FG plates with arbitrary boundary conditions using a series method. Hu and Zhang [22] also adopted the CPT with von Karman assumptions to perform vibration and stability analyses of FG plates under in-plane excitation. Free vibration of FG plates with various boundary conditions resting on an elastic foundation was investigated by Chakraverty and Pradhan [23] using the CPT and Rayleigh–Ritz method. Chakraverty and Pradhan [24] improved their previous work [23] by accounting for the effect of thermal environments. Ruan and Wang [25] investigate the vibration and stability of moving FG skew plates using the CPT and DQM.

The CPT was also used to analyze circular plates. For example, Ma and Wang [26] investigated the nonlinear bending and thermal postbuckling behaviors of FG circular plates under mechanical and thermal loadings. The governing equations derived in the framework of the CPT and von Karman assumptions were numerically solved using a shooting method. Li et al. [27] also studied the nonlinear postbuckling behavior of FG circular plate under mechanical and thermal loadings using the CPT with von Karman assumptions. The initial geometric imperfections of FG plates were taken into account in their study. Allahverdizadeh et al. [28] studied the steady-state vibration of FG circular plates in thermal environments using the CPT and a semi-analytical approach. Ghomshei and Abbasi [29] studied the axisymmetric thermal buckling of FG annular plates with variable thickness subjected to thermal loadings using the CPT and the finite element method.

In addition to FG plates, the CPT was also more preferably used for FG shells due to its simplicity. Loy et al. [30] studied the vibration of FG cylindrical shells with simply supported boundary conditions using the CPT and Rayleigh–Ritz method. A similar approach was adopted by Arshad et al. [31] to investigate the vibration characteristics of FG cylindrical shells under three different types of volume fraction laws. The vibration characteristics of