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Geometrically nonlinear transient analysis of thick deep composite curved beams with generalized differential quadrature method

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ABSTRACT

This article focuses on geometrically nonlinear transient analysis of thick deep laminated composite curved beams with generalized differential quadrature method. Generalized differential quadrature method is coupled with the weak form of the equation of motion. Virtual work principle is used to derive the equation of motion. Spatial derivatives in the equation of motion are expressed with generalized differential quadrature method. Geometric nonlinearity is considered through Green–Lagrange nonlinear strain–displacement relations that are derived using elasticity theory equations. First-order shear deformation theory is used to consider the transverse shear effect. Time integration of the equation of motion is carried out using Newmark average acceleration method. Several problems from the literature are solved with the proposed method and results are compared.

1. Introduction

Curved beams have wide applications in civil, mechanical and aerospace industries as load carrying structural members or stiffeners. Therefore, analyzes of these structures under variety of loading conditions are important to enable safe and economical designs. Several books were published on static, stability and dynamic behavior of beam structures [1–7]. Finite element method (FEM) and finite difference method (FDM) are commonly employed numerical methods in the analyzes of beam and other engineering problems [8–26]. However, both FEM and FDM typically use low-order schemes. Therefore, high accuracy with these methods is usually achieved with costly calculations.

Bellman et al. [27,28] introduced differential quadrature method (DQM) in early 1970s as an efficient alternative method, using less grid points with acceptable accuracy. Accuracy of DQM depends on the calculation of weight coefficients and the choice of grid points. In the original formulation of DQM, weight coefficients was calculated by an ill-conditioned Vandermonde type algebraic equation system limiting the use of large grid numbers. Later, Shu [29] developed simple explicit formula to calculate weight coefficients with arbitrary number of grid points leading to generalized differential quadrature (GDQ) method. Early applications of GDQ method were limited to regular domain problems. Civan and Sliepcevich [30] presented the domain decomposition technique with regular elements to solve pool boiling cavities. In the domain decomposition technique, problem domain is divided into a certain number of sub-domains or elements before the GDQ discretization is carried out on each sub-domain. GDQ method was extended to irregular domains by Lam [31]. The method was applied to the solution of thermal and torsional problems with geometry ranging from quadrangles to curved shapes. Domain decomposition technique with the use of GDQ method in each sub-domain is often referred to as the differential quadrature element method (DQEM) [29]. More general form of the domain decomposition technique includes the mapping technique, which is used to map a generic element onto a simple computational element. The general form combines the advantages of both GDQ method and FEM is often referred to as generalized differential quadrature element method (GDQE) [32]. Recently GDQ method was also included in the weak form solution of differential equations in which derivatives of field variables are calculated with GDQ method [33–36]. This method is called as weak form quadrature element method [35]. More detailed and complete information about the evolution of GDQ method and its application in various forms in the solution of engineering problems can be found in the related books and review articles [26,29,37–40].

Differential quadrature method and its improved forms were also used in static, vibration and buckling analysis of beam structures. A short review on the use differential quadrature method

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in the solution of beam problems in direct or coupled forms are given as following. He and Zhong [41] used the weak-form quadrature element method for large deflection elasto-plastic analysis of frames. Xiao and Zhong [42] performed non-linear analysis of planar frames based on geometrically exact beam theory using quadrature element. Chen [43] solved out of-plane deflection of non-prismatic curved beam structures by DQEM. Kang et al. [44] analyzed the out-of-plane static behavior of a curved shaft subjected to end torques, based on the curved-beam versions of the Euler and Timoshenko beam theories. Hajianmaleki and Qatu [45] investigated static and vibration analyzes of thick, generally laminated composite deep curved beams with different boundary conditions. For boundary conditions other than simply supported, GDQ method is used to solve the equations. Bert and Kang [46] used differential quadrature method for stress analysis of closely-coiled helical springs. Malekzadeh [47] employed two-dimensional layerwise-differential quadrature approach for the static analysis of thick laminated composite circular arches. Zhang and Zhong [48] used the weak form quadrature element method for the analysis of spatial geometrically exact shear-rigid beams. Jin and Wang [49] used the weak form quadrature element method for accurate free vibration analysis of Euler functionally graded beams. Viola et al. [50] coupled differential quadrature method with a domain decomposition technique for vibration analysis of damaged circular arches. Chen [51,52] employed DQEM for in-plane and out-of-plane vibration of curved beam structures considering the effect of shear deformation. Torabi et al. [53] used a DQEM for transverse vibration analysis of multiple cracked non-uniform Timoshenko beams with general boundary conditions. Malekzadeh and Setoodeh [54] carried out in-plane free vibration of laminated moderately thick circular deep arches using differential quadrature method. Malekzadeh et al. [55] used a hybrid layerwise and differential quadrature method for in-plane free vibration of laminated thick circular arches. Chen et al. [56] used elasticity solution for free vibration of laminated beams. In this study, the conventional state space method was combined with DQM to solve frequency equations. Chen et al. [57] used state-space-based differential quadrature method for free vibration analysis of generally laminated beams. Li and Shi [58] utilized state-space-based differential quadrature method for free vibration of a functionally graded piezoelectric beam. Rajasekaran [59] conducted free vibration of centrifugally stiffened axially functionally graded tapered Timoshenko beams using differential transformation and quadrature methods. Karami and Malekzadeh [60] investigated in-plane free vibration analysis of circular arches with varying cross-sections using differential quadrature method. Liu and Wu [61] investigated in-plane vibration analyses of circular arches by the GDQ rule. Ansari et al. [62] performed nonlinear forced vibration analysis of functionally graded carbon nanotube-reinforced composite Timoshenko beams using generalized differential quadrature method. Malekzadeh et al. [63] investigated out-of-plane free vibration of functionally graded circular curved beams in thermal environment using differential quadrature method. Moradi and Moghadam [64] performed vibration analysis of cracked post-buckled beams using differential quadrature method. Nassar et al. [65] used differential quadrature method for vibration analysis of structural elements. Shen and Zhong [66] performed static and vibrational analysis of partially composite beams using the weak-form quadrature element method. Karami and Malekzadeh [67] used differential quadrature element method to study static, vibration and buckling of beam structures. Eftekhari and Khani [68] coupled finite element method with differential quadrature element method to study dynamic behavior of beam under moving load. Du et al. [69] employed GDQ method to study vibration analysis of Euler beam. Pradhan and Murmu [70] used DQM for thermo-mechanical vibration of functional graded and sandwich beams.

From literature review it is seen that applications of differential quadrature method and its improved variants to dynamic beam problems are mostly devoted to vibration problems. Currently to the author's knowledge no study is available on geometrically nonlinear transient analysis of composite deep curved beams with GDQ method. Therefore, in this study GDQ method is applied to predict the nonlinear transient response of deep thick composite curved beams. GDQ method is coupled with the weak form of the equation of motion. Virtual work principle is used to derive the equation of motion. Spatial derivatives in the equation of motion are expressed with generalized differential quadrature method. Green–Lagrange nonlinear strain–displacement relationships are used to represent geometric nonlinearity and they are derived for deep curved beams using elasticity theory equations. First-order shear deformation theory (FOST) is used to take transverse shear effect into account. Newmark average acceleration method is used for the time integration of the equation of motion. Several problems from the literature are solved with the proposed method and results are compared.

2. Curved beam equations

2.1. Constitutive equations

Nonlinear strain–displacement relations in any three dimensional elastic body in an orthogonal curvilinear coordinate system α_1, α_2, ζ are given in Kundu et al. [71]. From Kundu et al. assuming zero displacement in α_2 direction ($v = 0$), nonlinear *longitudinal* strain in α_1 direction (ε_{11}) and shear strain in 1–3 plane (γ_{13}) are expressed as

$$\begin{aligned}\varepsilon_{11} &= e_{11} + \frac{1}{2}(e_{11}^2 + e_{31}^2)\\ \gamma_{13} &= e_{13} + e_{31} + e_{11}e_{13}\end{aligned} \tag{1}$$

where for constant radius of curvature $e_{11,}e_{31,}e_{13}$ are given as

$$\begin{aligned}e_{11} &= \frac{1}{(1+\zeta/R_1)}\left(\frac{\partial u}{\partial \alpha_1} + \frac{w}{R_1}\right)\\ e_{31} &= \frac{1}{(1+\zeta/R_1)}\left(\frac{\partial w}{\partial \alpha_1} - \frac{u}{R_1}\right)\\ e_{13} &= \frac{\partial u}{\partial \zeta}\end{aligned} \tag{2}$$

In Eq. (2) u and w indicate displacements in α_1 and ζ direction respectively. R_1 indicates principal radius of curvature in $\alpha_1 - \zeta$ plane.

Hereafter for simplicity in notation curvilinear coordinate system of α_1, $\alpha_{2,}\zeta$ is renamed as x, y, z and subscript of radius of curvature R_1 is dropped to be as R. Curved beam parameters are shown in Fig. 1.

In curved beam, displacements at a general point (x, z) at time t can be stated in terms of mid-plane displacements and rotation as

$$\begin{aligned}u(x,z,t) &= u_0(x,t) + z\theta_x(x,t)\\ w(x,z,t) &= w_0(x,t)\end{aligned} \tag{3}$$

where u_0, w_0 are mid-plane displacements and θ_x is the rotation about y axes.

After substituting Eqs. (2) and (3) into Eq. (1) Green–Lagrange nonlinear strain–displacement relationships for deep curved beams are obtained as following: