Composite Structures 127 (2015) 69–76

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A novel approach for displacement and stress monitoring of sandwich structures based on the inverse Finite Element Method

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ARTICLE INFO

Article history:
Available online 9 March 2015

Keywords:
Sandwich structures
Shape sensing
Stress sensing
Inverse Finite Element Method

ABSTRACT

The real-time reconstruction of the displacement and stress fields from discrete-location strain measurements is a fundamental feature for monitoring systems, which is generally referred to as shape- and stress-sensing. Presented herein is a computationally efficient shape- and stress-sensing methodology that is ideally suited for applications to laminated composite and sandwich structures. The new approach employs the inverse Finite Element Method (iFEM) as a general framework and the Refined Zigzag Theory (RZT) as the underlying plate theory. Using a C^0-discretization, a three-node inverse plate finite element is formulated. The element formulation enables robust and efficient modeling of plate structures instrumented with strain sensors that have arbitrary positions. The methodology leads to a set of linear algebraic equations that are solved efficiently for the unknown nodal displacements. These displacements are then used at the finite element level to compute full-field strains and stresses that may be in turn used to assess structural integrity. Numerical results for multilayered, highly heterogeneous laminates demonstrate the unique capability of this new formulation for shape- and stress-sensing.

1. Introduction

The use of composite and sandwich material systems as primary structures grew significantly during the past few decades with applications in civil and military aircrafts, launch vehicles, wind turbine blades and marine structures [1–3]. Despite their numerous advantages, composite structures may experience such modes of failure as delamination, face/core debond and impact damage [4], which can affect their load carrying capabilities. Since these types of damage are often barely visible (or even non visible) and hard to detect [5], the development of efficient and reliable integrated Structural Health monitoring systems that can predict possible damages become an issue of primary importance.

An efficient way to monitor a structural system is to use a network of in-situ strain sensors and measured strains to extrapolate the full-field displacements and stresses, thus enabling real-time damage predictions by means of appropriate failure criteria. This technology is commonly referred to as *shape- and stress-sensing*. Furthermore, the real-time evaluation of the deformed shape is also a vital technology for the development of composite smart structures such those of *morphed* capability and those with embedded conformal antennas that require real-time shape sensing to provide feedback for their actuation and control systems [6,7].

For composite and sandwich structures, the use of embedded optical-fiber networks represents an attractive technology for discrete-location strain measurements that can give rise to a large amount of strain data. Particularly, Fiber Bragg Grating (FBG) strain sensors are widely used due to their lightness, accuracy and ease to embed. Moreover, significant technological progresses have been made in embedding FBG sensors within composite and sandwich structures during the manufacturing process [8–12], thus making FBG sensors suitable for usage in operational conditions.

Various shape-sensing strategies have been explored in the literature for both plate and shell or frame structures. In a series of works [13–16], the measured strains are fitted by using an *a priori* set of basis functions and proper weights. Strain–displacement relationships are then used to evaluate the displacement field. Both global or piece-wise continuous basis functions were employed. In [15] Fiber-Bragg Grating (FBG) measured strains were fitted with a cubic polynomial. The strain field was then integrated to obtain plate deflection according to classical bending assumptions. Since the aforementioned methods make use of the Euler–Bernoulli beam hypotheses or the Kirchhoff plate hypotheses to define the strain–displacement relationships, they can only be

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http://dx.doi.org/10.1016/j.compstruct.2015.02.081
0263-8223/

applied to slender beams or thin plates. Todd and Vohra [16] showed how the shear effect can be included for the beam problem without requiring an independent measure of the shear strain.

Foss and Haugse [17], and Pisoni et al. [18] independently developed a modal method which employs normal modes to reconstruct the structural deformed shape. The unknown weights to be assigned to each normal mode are determinsed using strain–displacement relationship and measured surface strains. In [17,18] the mode shapes were experimentally estimated. In other works based on the modal method, analytical [19] or FEM generated mode shapes [20–24] were adopted. Although the experimental evaluation of the mode shapes can be onerous, it has the advantage of not requiring any knowledge of the material properties. Lively et al. [23] showed that when only lower natural modes are used the results can be inaccurate due to the aliasing of higher structural modes. In [24] it is pointed out that for high-frequency excitations a large number of natural vibration modes is needed to improve the accuracy, thus requiring a computationally intensive eigenvalue analysis and a large number of strain measurements.

To reconstruct the deformed shape of shear-deformable plate and shell structures, Tessler and Spangler [25,26] developed an inverse Finite Element Method (iFEM). The proposed methodology employs a weighted-least-square variational principle and accounts for the complete set of First-order Shear Deformation Theory (FSDT) modes. Since only strain–displacement relations are used in the formulation, both the static and the dynamic responses can be reconstructed without any *a priori* knowledge of material, inertial, loading, or damping structural properties. By using C^0-continuous kinematic approximations, they developed a three-node inverse shell element called iMIN3 [25]. The variational principle of the iFEM was specialized by Gherlone et al. [27] to the shape sensing of shear-deformable beam and frame structures. Several numerical and experimental tests showed that iFEM is versatile and robust [27–29]. A similar approach was used in [30], where compatibility between analytical and measured bending curvatures of the Kirchhoff plate theory is enforced in a weighted-least-squares sense. The weighting coefficients were adjusted in order to account for the inherent errors in the measured strain data. The weights were computed for a given data-acquisition apparatus, load case and test article, with the consequent difficulties in generalizing the procedure.

All the existing shape-sensing methods adopt First-order Shear Deformation Theory (FSDT) and have been exclusively applied to homogeneous or nearly-homogeneous beams or plates. Although generally regarded as an accurate theory, FSDT may lead to somewhat inadequate predictions when applied to relatively thick composite and sandwich structures. For such structures, higher-order equivalent-single-layer theories [31] provide improved predictions, specifically for the global response quantities such as deflection and natural frequencies; nevertheless, even these theories fail to predict through-the-thickness distributions of displacements, strains, and stresses with sufficient accuracy. Layer-wise theories [32] usually lead to highly accurate response predictions; however, these are obtained at the expense of computational efficiency and modeling complexity, especially for multilayered structures, since the number of unknowns depends on the number of material layers. A good compromise between adequate accuracy and computational efficiency are the so-called *zigzag* theories, first developed by Di Sciuva [33–41]. For these theories the number of kinematic variables is the same regardless of the number of material layers. Furthermore, the zigzag displacement field is able to model the cross-sectional distortion that is typical of multilayered composite and sandwich structures. Recently, a Refined Zigzag Theory has been developed [42,43] which allows more accurate predictions of the response quantities, including the transverse shear stresses, and a more efficient computation by means of C^0-continuous finite elements (instead of C^1-continuous finite elements required by the previous zigzag theories).

In this paper, the Tessler–Spangler [25,26] inverse Finite Element Method (iFEM) is reformulated to include the kinematic assumptions of the Refined Zigzag Theory (RZT) [42,43]. The new formulation is thus intended for applications dealing with multilayered composite and sandwich structures possessing a high degree of anisotropy and heterogeneity. The variational principle is then discretized using a C^0-continuous three-node inverse plate finite element. Numerical results are presented for moderately thick sandwich laminates subjected to various boundary and loading conditions. Finally, superior stress-sensing capabilities of the present formulation are demonstrated for a select set of challenging material systems. This paper is an extended and enhanced version of the work presented in [44], including also new numerical results investigating influence of the plate slenderness and the core-to-face thickness ratio, and the sensitivity to input data.

2. Kinematic assumptions of the Refined Zigzag Theory for plates

Herein the kinematic assumptions of the Refined Zigzag Theory (RZT) for plates are briefly reviewed. In particular, the strain field is formally re-written in order to define the *strain measures* to be used in the iFEM variational formulation (see Section 3).

Consider a plate of thickness $2h$ made of N perfectly bonded orthotropic material layers (see Fig. 1); the superscript (k) denotes the kth layer. The plate is referred to a Cartesian coordinate system (x_1, x_2, z) where (x_1, x_2) are the in-plane coordinates and z is the thickness coordinate that ranges from $-h$ to h, with $z = 0$ identifying the mid-plane and $z_{(j)}$ identifying the jth interface (see Fig. 2).

The displacement field of RZT for plates is [43]

$$\begin{aligned} u_1^{(k)}(x_1,x_2,z) &\equiv u(x_1,x_2) + z\,\theta_1(x_1,x_2) + \phi_1^{(k)}(z)\,\psi_1(x_1,x_2) \\ u_2^{(k)}(x_1,x_2,z) &\equiv v(x_1,x_2) + z\,\theta_2(x_1,x_2) + \phi_2^{(k)}(z)\,\psi_2(x_1,x_2) \\ u_z(x_1,x_2,z) &\equiv w(x_1,x_2) \end{aligned} \quad (1)$$

where $u_1^{(k)}$ and $u_2^{(k)}$ are the in-plane displacements and u_z is the transverse displacement. RZT has seven kinematic variables, $\mathbf{u} = \{u, v, w, \theta_1, \theta_2, \psi_1, \psi_2\}^T$. The same definition of the FSDT yields for the first five components of the vector $\mathbf{u}$; namely, u, v, and w are the uniform displacement components along the x_1, x_2, and z-axis respectively, whereas θ_1 and θ_2 are the average rotations of the transverse normal around the positive x_2-axis and the negative x_1-axis, respectively. The RZT variables $\psi_\alpha(\alpha = 1, 2)$ are the amplitudes of the *zigzag* contributions to the in-plane displacement in the x_α-directions (see Fig. 1). The zigzag terms $\phi_\alpha^{(k)}\psi_\alpha(\alpha = 1, 2)$ in Eq. (1) describe the C^0-continuous cross-sectional distortions that are typical of multilayer laminates. The zigzag functions, $\phi_\alpha^{(k)}(z)$, have units of length and are piecewise linear, C^0-continuous functions of the thickness coordinate and of the transverse shear moduli of the laminate layers (see [43]).

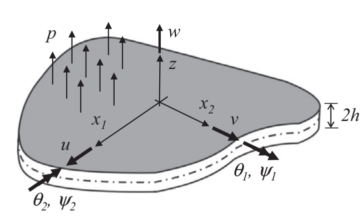

Fig. 1. Plate notation.