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Size-dependent nonlinear bending and postbuckling of functionally graded Mindlin rectangular microplates considering the physical neutral plane position

R. Ansari [a], R. Gholami [b,*], M. Faghih Shojaei [a], V. Mohammadi [a], M.A. Darabi [a]

[a] Department of Mechanical Engineering, University of Guilan, P.O. Box 3756, Rasht, Iran
[b] Department of Mechanical Engineering, Lahijan Branch, Islamic Azad University, P.O. Box 1616, Lahijan, Iran

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ABSTRACT

In this paper, the nonlinear bending and postbuckling characteristics of Mindlin rectangular microplates made of functionally graded (FG) materials are studied based on the modified couple stress theory (MCST). This theory facilitates considering size dependency through introducing material length scale parameters. The FG microplates, whose volume fraction is expressed by a power law function, are considered to be made of a mixture of metals and ceramics. By considering the physical neutral plane position, the stretching–bending coupling is eliminated in both nonlinear governing equations and boundary conditions of FG microplates. With the aid of MCST and the principle of virtual work, the governing equations and corresponding boundary conditions are derived. Then, the obtained governing equations and boundary conditions are discretized through the generalized differential quadrature (GDQ) method. Finally, the resulting nonlinear parameterized equations are solved by the pseudo-arclength continuation technique. The effects of material gradient index, length scale parameter, length-to-thickness ratio, and boundary conditions on the nonlinear bending and postbuckling responses of FG microplates are investigated.

1. Introduction

Beams and plates at the scale of microns and sub-microns are among the chief structures extensively used in micro/nano-electro-mechanical systems (MEMS/NEMS) and atomic force microscopes [1–5]. Some experiments have demonstrably shown the size-dependent torsion and bending behaviors of microbeams [6–9] and it has been found that size-dependent behavior is an inherent property that has considerable effects when the thickness of beam is comparable to the internal material length scale parameter.

Since the classical elasticity theory fails to consider scale dependency, several attempts have been made to develop unconventional theories capable of accommodating the size dependence of micro- and nanostructures. In this respect, one can mention the following non-classical continuum theories: the nonlocal elasticity, the couple stress elasticity, the strain gradient elasticity and the surface elasticity theories [10–13]. These theories have been broadly employed to examine the mechanical responses of micro- and nanostructures [14–21]. The couple stress theory describes the microstructure-dependent size effects by introducing two material length scale parameters [10,22]. This theory was then modified by Yang et al. [23] and named the modified couple stress theory (MCST). They precipitated incorporating the size effect by considering only one material length scale parameter in addition to classical material constants. MCST was used to develop several size-dependent models such as non-classical Euler–Bernoulli microbeam, Timoshenko microbeam and Kirchhoff microplate models. Some investigators have presented analytical solutions for the static and dynamic behaviors of size-dependent microplates based on MCST in the framework of Kirchhoff microplate model [24–26]. Also, Ke et al. [27] studied the free vibration behavior of Mindlin microplates based on MCST.

The superlative characteristics of FG materials have motivated researchers to employ them in MEMS and NEMS to acquire high sensitivity and desired performance. Recently, MCST has been employed to develop the size-dependent FG microbeam models. In this direction, several investigations on the nonlinear vibration, static buckling, bending and dynamic stability of FG Euler–Bernoulli and Timoshenko microbeams have been conducted [28–31]. In the aforementioned studies, it is considered that the

* Corresponding author. Tel./fax: +98 141 2222906.
E-mail address: gholami_r@liau.ac.ir (R. Gholami).

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undeformed plane (the physical neutral plane) is located at the physical middle plane. Since the material properties of FG micro- and nanostructures in the thickness direction are inhomogeneous, their physical neutral plane is not identical to their geometric middle plane. This issue has attracted the attention of some researchers. For example, Asghari and his associates [32] studied the bending and free vibration of FG microbeams by considering the physical neutral plane. The nonlocal bending, buckling and vibration problems of FG nanobeams are addressed by Eltaher et al. [33] based on the physical neutral plane concept. Ke et al. [34] investigated the axisymmetric postbuckling of FG annular microplates by taking the physical neutral plane position into account. The results of these works revealed that considering physical neutral plane can remove the stretching–bending coupling in FG structures.

It should be noted that in the case of FG microplates with simply-supported boundary conditions under in-plane edge compressive loads or temperature change, the bifurcation buckling does not happen because of the stretching–bending coupling effect [35,36]. Hence, when there is no compressive load applied on the physical neutral surface, the bifurcation solutions for unsymmetric simply-supported FG microplates under the in-plane compression and temperature change may be wrong from the physical standpoint. Motivated by this consideration, it is supposed that the compressive in-plane loads are acted on the physical neutral surface of FG microplates. However, it should be remarked that in the case of clamped FG microplates, the buckling loads are present and the hypothesis of mid-plane symmetric structure is not required.

This paper investigates the postbuckling of rectangular FG microplates considering the physical neutral plane position based on MCST. To accomplish this aim, first, a size-dependent Mindlin plate model capable of considering small scale effects is developed for FG rectangular microplates including the von Kármán geometric nonlinearity. The volume fraction of FG materials is expressed by a power law function. By considering the physical neutral plane position, the stretching–bending coupling is eliminated in FG microplates. The principle of virtual work is used to derive the governing equations and associated boundary conditions. Afterwards, the obtained nonlinear governing stability equations are discretized through the GDQ method, and then solved via the pseudo-arclength continuation technique. The effects of important parameters including material gradient index, length scale parameter, length-to-thickness ratio, and boundary conditions on the nonlinear bending and postbuckling analysis of FG microplates are investigated.

2. Modified couple stress theory

According to MCST [23], the stored strain energy U_m in a continuum made of a linear elastic material occupying region Ω with infinitesimal deflections can be written as

$$U_m = \frac{1}{2}\int_{\Omega}\left(\sigma_{ij}\varepsilon_{ij} + m^s_{ij}\chi^s_{ij}\right)dv \tag{1}$$

in which the components of strain tensor and the symmetric curvature tensor symbolized by ε_{ij}, and χ^s_{ij}, respectively, are given as

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{i,j}u_{j,i}\right), \tag{2a}$$

$$\chi^s_{ij} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right), \quad \theta_i = \frac{1}{2}\left(curl(\mathbf{u})\right)_i. \tag{2b}$$

where u_i stands for the components of displacement vector $\mathbf{u}$ and θ_i shows the infinitesimal rotation vector $\boldsymbol{\theta}$.

For a linear isotropic elastic material, the constitutive equation can be characterized in terms of the kinematic parameters effective on the strain energy density as [37,38]

$$\sigma_{ij} = \lambda tr(\varepsilon)\delta_{ij} + 2\mu\varepsilon_{ij}, \quad m^s_{ij} = 2\mu l^2\chi^s_{ij}. \tag{3}$$

The parameters σ_{ij} and m^s_{ij} are known as the components of the symmetric part of stress tensor $\boldsymbol{\sigma}$ and the deviatoric part of the couple stress tensor $\boldsymbol{m^s}$, respectively; the symbol δ refers to the Kronecker delta and l is the material length scale parameter. Also, the parameters λ and μ appearing in the constitutive equation of the classical stress $\boldsymbol{\sigma}$ represent two Lamé's constants and are defined for the plane stress state as

$$\lambda = \frac{Ev}{1-v^2}, \quad \mu = \frac{E}{2(1+v)}. \tag{4}$$

in which E and v are Young's modulus and Poisson's ratio, respectively.

3. Governing equations and corresponding boundary conditions

An FG rectangular microplate made of a mixture of ceramics and metals with length a, width b and thickness h subjected to the in-plane forces N^0_{xx}, N^0_{yy} and N^0_{xy} and applied transverse load Q_0 is considered. The microplate is assumed to be metal-rich and ceramic-rich at the bottom surface $(z = -h/2)$ and top surface $(z = h/2)$, respectively. Also, the effective material properties of FG microplate including Young's modulus E and Poisson's ratio v can be estimated as

$$E(z) = E_c V_c + E_m V_m, \quad v(z) = v_c V_c + v_m V_m. \tag{5}$$

where the subscripts m and c represents metal and ceramic phases, respectively; V denotes the volume fraction of material phases.

Among all functions available for expressing the volume fraction variation of microplate's components, the power law function is employed herein as follows [39]

$$V_c(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^k, \quad V_m(z) = 1 - \left(\frac{1}{2} + \frac{z}{h}\right)^k \tag{6}$$

where k is the volume fraction exponent or material gradient index. Based on the first-order shear deformation plate theory, the in-plane displacements can be characterized as the linear functions of plate thickness, and the transverse deflection is considered to be constant through the plate thickness. Therefore, the displacement field in a Mindlin plate can be described as

$$u_x = u(x,y) - \bar{z}\psi_x(x,y), \quad u_y = v(x,y) - \bar{z}\psi_y(x,y), \quad u_z = w(x,y). \tag{7}$$

in which $u(x,y)$ and $v(x,y)$ are mid-plane displacements, and $w(x,y)$ represents the lateral deflection of microplate; and ψ_x and ψ_y denote the rotations of the transverse normal about y- and x-axis, respectively. Moreover, one has $\bar{z} = z - z_0$ in which z_0 stands for the z-coordinate of the physical neutral plane that can be calculated as $z_0 = \int_{-h/2}^{h/2}(\lambda + 2\mu)zdz / \int_{-h/2}^{h/2}(\lambda + 2\mu)dz$. This means that the neutral axial is dependent on the material distribution in the thickness direction. Accordingly, for inhomogeneous FG microplates, the physical neutral plane is not equal to the geometric middle plane. By taking the physical neutral surface as the reference plane, it is possible to ignore the stretching–bending coupling stiffness component.

Based on the von Kármán hypothesis and by inserting Eq. (7) into (2a), the nonzero components of strain–displacement equations are obtained as

$$\begin{aligned} &\varepsilon_{xx} = \phi_0 - \bar{z}\phi_1, \quad \varepsilon_{yy} = \varphi_0 - \bar{z}\varphi_1, \quad \varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}(\kappa_0 - \bar{z}\kappa_1), \\ &\varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2}\gamma_1, \quad \varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2}\gamma_2. \end{aligned} \tag{8}$$