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Geometrically nonlinear static FE-simulation of multilayered magneto-electro-elastic composite structures

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ABSTRACT

A fully geometrically nonlinear finite rotation shell element for static analysis of layered magneto-electro-elastic (MEE) coupled composite structures is proposed. Reissner–Mindlin first-order shear deformation (FOSD) theory with full geometrically nonlinear strain–displacement relations and finite rotations is used to obtain the variational formulation. The scalar electric and magnetic fields are assumed along with the quasi-static behavior of MEE layers. Electric and magnetic potentials are assumed to vary quadratically in the transverse direction. Three linear constitutive relations are used describe the magneto-electro-elastic coupling. The magneto-electro-elastic composite four node shell element behavior is refined by embodying an assumed natural strain (ANS) method for transverse shear strains, and an enhanced assumed strain (EAS) method for in-plane bending strains. The developed finite element model is deployed for static analysis of layered MEE structures in sensor and actuator configurations, and results are compared with the available references. Additionally, several numerical examples are simulated to show the potentiality and predictive capabilities of the proposed finite element method.

1. Introduction

The traditional single phase materials (piezoelectric, magnetostrictive etc) except in few cases [1] show either electromechanical or magnetomechanical coupling without any magneto-electric coupling. In recent years, a new class of advanced high-performance materials like magneto-electro-elastic (MEE) materials have been developed to act as sensors and actuators to inspect and control the response of structures. Because of the ability of converting energies between elastic, electric and magnetic form, these multi-purpose MEE materials/structures en route new impressive and ingenious applications [2,3] in many areas, including aerospace, robotics, automotive, biomedical and next generation transport vehicles. Multiphase MEE materials are obtained either by combining piezoelectric and piezomagnetic particles/fibers in bulk form [4] or laminated form [5] (i.e. stacking piezoelectric and piezomagnetic layers) to accomplish required full magneto-electro-mechanical coupling. Research on studying the behavior of MEE layers embedded into the laminated composites has gained significant attention in very recent years.

An early pioneering work of 3-D analytical solutions of MEE plates under simply supported boundary conditions was presented by Pan [6], later extended to study free vibrations [7] and cylindrical bending [8] by Pan and co-workers. Wang et al. [9] studied exact solutions of MEE plates by state space approach. Wu et al. [10] investigated Pagano [11] solution for three dimensional simply supported functionally graded MEE rectangular plates. Tsai and Wu [12] studied dynamic response of functionally graded MEE shells with open circuit boundary conditions. However, these analytical results are limited to specific boundary conditions and geometries. To overcome this limitations and to deal with complex configurations numerical solution methods are required. Furthermore, because of the computational costs involved in 3-D finite element simulations for laminated plates and shells, 2-D finite elements have been proposed to reduce the computational effort by retaining the accuracy level. Sunar et al. [13] developed finite element for thermopiezomagnetic smart beams by using variational approach with the aid of thermodynamic potential. Moita et al. [14] proposed a layerwise finite element to analyze MEE plates. Carrera and Nali [15] presented finite element solutions for multilayered MEE plates by considering electric and magnetic primary variables as independent variables with layerwise modeling. Kumaravel et al. [16] investigated a rectangular magneto-electro-elastic strip under thermal

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loads. Daga et al. [17] analyzed transient response of multiphase magneto-electro-elastic sensors using 3D elements with different MEE materials. Recently, Milazzo et al. [18] proposed an equivalent single layered MITC plate element to analyze MEE multilayered plates by condensing the electromagnetic state into mechanical primary variables and also by neglecting in-plane components of electric and magnetic field.

The above mentioned literature survey indicates that the proposed finite element formulations are capable of dealing with geometrically linear solutions (i.e small displacements). Concerning the geometrically nonlinear large deflection behavior, the literature found to be very scarce despite the fact that these MEE structures are flexible and undergo deformations larger than thickness which has significant impact in sensor and actuator applications. Xue et al. [19] investigated analytical solutions of rectangular MEE thin plates at large deflections with the approximation of linear electric and magnetic potentials over the thickness in the framework of Kirchoff-Love hypothesis using von Kármán type nonlinearity. Sladek et al. [20] presented meshless local Petrov–Galerkin method to analyze the large deflections of MEE plates by adopting von Kármán type nonlinear strain–displacement relations based on Reissner–Mindlin theory. Most recently, Milazzo et al. [18] extended the small deflection MITC plate element for multilayered MEE plates to large deflections [21] by considering von Kármán type nonlinearity in the first-order shear deformation (FOSD) formulation. Ray and Kattimani [25,26] developed a solid element for vibration damping of MEE plates and shells using active constrained piezopatches based on von Kármán type nonlinearity in FOSD formulation and with the assumption of linear electro-magnetic potentials through the thickness. Layerwise and equivalent single layer models are developed for MEE plates by Milazzo [22] and later the same author extended to free vibration analysis with refined equivalent single layer approach [23] and also to higher order model by assuming fourth order strains through-thickness distribution [24]. However, the recent papers on large deflections are limited to either plates [25,20,19,18] or simplified assumptions of field variables like von Kármán type nonlinear theories [26,21–24] and linear assumption of electro-magnetic potentials [19,25,26].

It is worth to note, that so far there is no available literature on fully geometrically nonlinear analysis and 2-D shell element for smart MEE layered plates and shell structures. However, it is important to observe the mechanical behavior of such structures more accurately at large deformations, for actual control and sensor applications, in order to design the appropriate control system. By observing fundamental equations of electromagnetic media in shells/laminates [27] and the state of art in modeling, here a finite element formulation of fully geometrically nonlinear magneto-electro-elastic coupled shell theory is presented in order to accomplish quasi-static analysis of MEE layered plates/shells. The formulation is based on the assumptions of small strains and finite rotations in the framework of Reissner–Mindlin FOSD hypothesis. The finite rotation theory (FRT) developed by Wagner and Gruttmann [28] and applied by Lentzen [29] is used to develop finite elements for composite plates and shells integrated with magneto-electro-elastic (MEE) layers. The present shell element has 5 mechanical DOFs and 3 electrical and magnetic DOFs per node. The strain field is enriched by assumed natural strain (ANS) and enhanced assumed strain (EAS) methods, respectively. The developed composite magneto-electro-elastic laminated shell element is validated by several numerical examples dealing with the quasi-static analysis of laminated composite structures bonded with MEE layers as sensors and actuators in the geometrically nonlinear range of deformation and comparisons are made with those reported in literature.

2. Basic equations and kinematics

2.1. Constitutive relations

The three linear constitutive equations for magneto-electro-elastic continuum according to the common practice of 2-D shell theories, are given by [27,21]

$$\boldsymbol{S} = \boldsymbol{c}\boldsymbol{\varepsilon} - \boldsymbol{e}^T\boldsymbol{E} - \boldsymbol{q}^T\boldsymbol{H}, \tag{1}$$

$$\boldsymbol{D} = \boldsymbol{e}\boldsymbol{\varepsilon} + \boldsymbol{\epsilon}\boldsymbol{E} + \boldsymbol{\kappa}\boldsymbol{H}, \tag{2}$$

$$\boldsymbol{B} = \boldsymbol{q}\boldsymbol{\varepsilon} + \boldsymbol{\kappa}\boldsymbol{E} + \boldsymbol{\mu}\boldsymbol{H}, \tag{3}$$

where $\boldsymbol{S}$ denotes the stress vector, $\boldsymbol{\varepsilon}$ the strain vector, $\boldsymbol{D}$ the electric displacement vector and $\boldsymbol{E}$ the electric field vector, $\boldsymbol{B}$ the magnetic induction vector and $\boldsymbol{H}$ the magnetic field vector:

$$\boldsymbol{S} = \begin{Bmatrix} S_{11} \\ S_{22} \\ S_{12} \\ S_{13} \\ S_{23} \end{Bmatrix}, \quad \boldsymbol{D} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}, \quad \boldsymbol{B} = \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix},$$
$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{Bmatrix}, \quad \boldsymbol{E} = \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}, \quad \boldsymbol{H} = \begin{Bmatrix} H_1 \\ H_2 \\ H_3 \end{Bmatrix}. \tag{4}$$

It should be noted that the considered structures are thin, thus the normal stress S_{33} is vanishing and not presented in Eq. (4). The $\boldsymbol{\varepsilon}, \boldsymbol{E}$ and $\boldsymbol{H}$ are the secondary field variables corresponding to the primary field variables, i.e. elastic displacement vector, electric potential and magnetic potential u, ϕ and ψ, respectively.

By assuming certain material symmetry in MEE materials and poling direction along the thickness (z-direction), the following matrices are given in plane-deformation problems (see [18–20]) as

$$\boldsymbol{c} = \left[\begin{array}{ccc|cc} c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 \\ \hline 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & c_{55} \end{array}\right] = \begin{bmatrix} [C_1] & 0_{(3\times2)} \\ 0_{(2\times3)} & [C_2] \end{bmatrix}, \tag{5}$$

$$\boldsymbol{e}^T = \left[\begin{array}{cc|c} 0 & 0 & e_{13} \\ 0 & 0 & e_{13} \\ 0 & 0 & 0 \\ \hline e_{41} & 0 & 0 \\ 0 & e_{41} & 0 \end{array}\right] = \begin{bmatrix} 0_{(3\times2)} & [\boldsymbol{e}_1] \\ [\boldsymbol{e}_2] & 0_{(2\times1)} \end{bmatrix}, \tag{6}$$

$$\boldsymbol{q}^T = \left[\begin{array}{cc|c} 0 & 0 & q_{13} \\ 0 & 0 & q_{13} \\ 0 & 0 & 0 \\ \hline q_{41} & 0 & 0 \\ 0 & q_{41} & 0 \end{array}\right] = \begin{bmatrix} 0_{(3\times2)} & [\boldsymbol{q}_1] \\ [\boldsymbol{q}_2] & 0_{(2\times1)} \end{bmatrix}, \tag{7}$$

$$\boldsymbol{\epsilon} = \left[\begin{array}{cc|c} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ \hline 0 & 0 & \epsilon_3 \end{array}\right] = \begin{bmatrix} [\boldsymbol{\epsilon}_1] & 0_{(2\times1)} \\ 0_{(1\times2)} & [\boldsymbol{\epsilon}_3] \end{bmatrix}, \tag{8}$$

$$\boldsymbol{\kappa} = \left[\begin{array}{cc|c} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ \hline 0 & 0 & \kappa_3 \end{array}\right] = \begin{bmatrix} [\boldsymbol{\kappa}_1] & 0_{(2\times1)} \\ 0_{(1\times2)} & [\boldsymbol{\kappa}_3] \end{bmatrix}, \tag{9}$$