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# A cutout isogeometric analysis for thin laminated composite plates using level sets

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ABSTRACT

Numerical modeling with treatment of trimmed objects such as internal cutouts in terms of NURBS-based isogeometric analysis presents several challenges, primarily due to the tensor product of the NURBS basis functions. In this paper we develop a new simple and effective isogeometric analysis for modeling buckling and free vibration problems of thin laminated composite plates with cutouts. We adopt the classical plate theory for the present formulation. The new approach can nicely overcome the drawbacks in modeling complex geometries with multiple-patches as the level sets are used to describe the internal cutouts; while the numerical integration is used only inside the physical domain. Numerical examples with complicated shapes are considered and analyzed to show the influences of cutout geometry, fiber orientation, boundary conditions, etc. on natural frequency and buckling behaviors of laminated plates. The results are compared with reference solutions showing a high accuracy of the proposed method.



## 1. Introduction

Owing to the advantages like light weight, longer life, and fatigue endurance etc., thin laminated composites have been widely used as components in engineering structures. Due to a large requirement of laminates for a variety of engineering applications, thin plate structures with arbitrary cutouts are inevitable. The presence of cutouts can significantly affect the dynamic and buckling behaviors of structures in particular, and their performance in general. Studies on the analysis of eigenvalues and stability of thin laminates with cutout are of great importance for many practical applications including airplane wing, fuselage and ribs.

Because of the complexity of the laminated composites with cutouts, numerical methods are extensively used in this subject. Studies on the vibration and buckling problems of thin laminated composite plates with cutouts have performed using different numerical approaches including finite element method (FEM) [1–4], Rayleigh–Ritz method [5], meshfree method [6], finite strip method [7,8] and extended finite element method (XFEM) [9]. In recent years, the isogeometric analysis (IGA) [10,11] has introduced and become popular since it inherently owns many great advantages including exact geometry representation, higher-order continuity, simple mesh refinement, and avoiding the traditional mesh generation procedure. Some insights into mathematical properties [12,13], integration method [14,15] and splines techniques [16] have been studied. The inherent characteristics of IGA make it superior to the classical FEM in many aspects [10]. The IGA has successfully applied to many engineering problems including plates and shells [17–20], fluid mechanics [21], fluid–structure interaction [22], damage and fracture mechanics [23], contact mechanics [24], unsaturated flow problem in porous media [25], and structural shape optimization [26].

Interesting features of exact geometry representation and high-order smoothness of NURBS basis functions have attractive for the analysis of plates and shells problems. The $C^1$ continuity requirement of classical plate theory is easily handled without additional efforts due to the high-order continuity of NURBS basis functions. Based on classical plate theory, a direct construction of rotation-free isogeometric shell was initially introduced in [27], and fully developed for multiple NURBS patches using the bending strip method by Kiendl [28]. Later, the rotation-free isogeometric shell element was extended to large deformation [29], free vibration and buckling analysis of laminated plates [30] and functional

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graded plates [31], cloth simulation [32]. Investigated in the previous works, it exhibits that the rotation-free isogeometric plate/shell elements can attain very good accuracy and are efficient for plate and shell structures. However, most of the applications are limited to single-patch structures and simply geometry. For complex geometries like structure with cutouts, the trimmed NURBS surface is useful. Shojaee et al. [30,33] artificially divided the square plate with a hole of complicated shape into several NURBS patches and then applied the bending strip method to maintain $C^1$ continuity between patches. This approach however is found to be ineffective and not straightforward to unify design and analysis. To deal with the trimmed NURBS surfaces, Schmidt et al. [34] alternatively developed a local reconstruction technique to rebuild trimmed elements with separate patches in which the geometric errors and loss of the higher-order continuity along the interior and limited to single trimming curves are introduced. Nevertheless, numerical modeling with treatment of trimmed objects such as internal cutouts in terms of NURBS-based IGA has some drawbacks primarily because of the tensor product induced by the NURBS basis. The existing geometric models with trimmed NURBS surface thus remain a challenging problem in computational mechanics community.

In order to conveniently deal with complex geometry problems, the IGA on one hand has combined with the enrichment method to form the so-called XIGA [35], and on the other hand that combines with the finite cell method (FCM) resulting the *finite cell extension to isogeometric analysis* [36,37], in which the level sets are used to define the geometry of the computational domain. In this work, the level sets are used to describe the internal cutouts and the numerical integration is used only inside the physical domain. One must be noted that most of the preceding studies using IGA with level sets were developed for elastoelastic problems, but have not accounted yet for dynamic and buckling problems of thin laminated composite structures with complicated shapes. The objective of this paper is to fill such a gap which essentially devotes to modeling dynamic and buckling of complicated thin laminated structures by a new cutout IGA approach. This new simple and effective IGA is found accurately and effectively, which will be demonstrated later in the numerical results, in modeling complicated thin laminated composites with cutouts based on the classical plate theory.

The rest of this paper is structured as follows: fundamental of NURBS basis functions and their derivatives are presented in Section 2. In Section 3, the IGA discrete equations based on classical plate theory and level sets for thin laminated composites with cutouts are derived. Numerical examples with complicated shapes for the free vibration problems of thin laminated structures with cutouts are presented in the next section to show the accuracy of the proposed approach. Similarly, buckling results for thin laminates with cutouts are presented in Section 5. We end with some conclusions drawn from the study in Section 6.

## 2. NURBS basis functions and their derivatives

The NURBS basis functions and their derivatives are briefly introduced in this section. For details, curious readers may refer to [10,38].

The NURBS is known as a generalization of B-splines. A B-spline is defined on a one dimensional parametric space $\xi \in [0,1]$, by a set of non-decreasing numbers called knot vector $\boldsymbol{k}(\xi) = \{\xi_1 = 0, \ldots, \xi_i, \ldots, \xi_{n+p+1} = 1\}^T (\xi_i \leqslant \xi_{i+1})$, and a set of control points $\boldsymbol{P}_i, i = 1, \ldots, n$, where $n$ and $p$ is the number of splines basis functions and the order of splines basis functions, respectively. The non-zeros knot span $[\xi_i, \xi_{i+1})$ is defined as element in IGA. A knot vector $\boldsymbol{k}(\xi)$ is called an open knot vector when the two ends of the knot are repeated $p+1$ times.

With a given knot vector $\boldsymbol{k}(\xi)$, the B-spline basis function, written as $N_{i,p}(\xi)$, is defined recursively as follows [38]:

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leqslant \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad \text{for } p = 0, \tag{1}$$

and

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \quad \text{for} \quad p \geqslant 1. \tag{2}$$

A quadratic B-spline basis functions example is presented in Fig. 1 using the following knot vector

$$\boldsymbol{k}(\xi) = \{\xi_1, \xi_2, \ldots \xi_9, \xi_{10}\}^T = \{0, 0, 0, 0.2, 0.4, 0.6, 0.8, 1, 1, 1\}^T \tag{3}$$

It can be observed that a B-Spline basis function is $C^\infty$ continuous inside a knot span, i.e., between two distinct knots, and $C^{p-1}$ continuous at a single knot. This character satisfies the requirement of the classical plate theory.

The NURBS basis function $R_{i,p}(\xi)$ in the framework of partition of unity is constructed by a weighted average of the B-spline basis functions [38] as follows:

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi) w_i}{\sum_{j=1}^{n} N_{j,p}(\xi) w_j} \tag{4}$$

where $w_i$ is the $i$th weight; the NURBS basis function is degenerate into B-spline basis function for $w_i = 1$.

Similarly, the bivariate NURBS basis function for a NURBS surface is given by

$$R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi) N_{j,q}(\eta) w_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi) N_{j,q}(\eta) w_{i,j}} = \frac{N_{i,p}(\xi) N_{j,q}(\eta) w_{i,j}}{W(\xi,\eta)} \tag{5}$$

where $w_{i,j}$ represents the 2D weight; $N_{j,q}(\eta)$ is the B-spline basis of order $p$ defined on the knot vector $\boldsymbol{k}(\eta)$, followed the recursive formula shown in Eqs. (1) and (2); $W(xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi) N_{j,q}(\eta) w_{i,j}$ is the weighting function for a NURBS surface.

It should be stated here that the NURBS basis functions own the same properties as B-splines. By using the NURBS basis functions, a NURBS surface of order $p$ in the $\xi$ direction and order $q$ in the $\eta$ direction can be constructed as follows:

$$\boldsymbol{S}(\xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} R_{i,j}^{p,q}(\xi,\eta) \boldsymbol{P}_{i,j}, \tag{6}$$

where $\boldsymbol{P}_{i,j}$ represents the coordinates of control points in two dimensions.

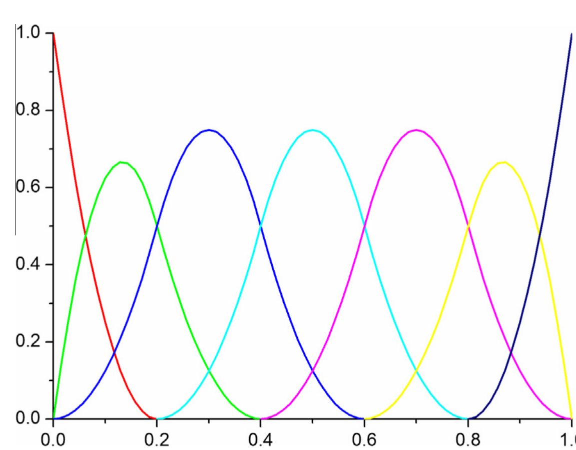


**Fig. 1.** Quadratic B-spline basis functions.