Composite Structures 127 (2015) 185–198

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A finite element formulation for smart piezoelectric composite shells: Mathematical formulation, computational analysis and experimental evaluation

Murilo Sartorato, Ricardo de Medeiros, Volnei Tita *

Department of Aeronautical Engineering, Engineering School of São Carlos, University of São Paulo, Av. João Dagnone 1100, São Carlos, SP, Brazil

ARTICLE INFO

Article history:
Available online 14 March 2015

Keywords:
Composite structure
Smart shell finite element
Piezoelectric materials
Dynamic tests

ABSTRACT

A formulation of a smart shell finite element for laminated curved structures with piezoelectric layers working under either the d31 or d33 effects has been developed in order to simulate dynamic tests. The electrical domain was modeled using a first order theory for the electrical field and a layer-wise approach. The mechanical domain is modeled using a degenerated shell theory with implicit curvature, considering a first order shear deformation theory and an equivalent single layer approach. The final formulation was implemented within Abaqus™ commercial finite element analysis package using its User Element (UEL) subroutine. First, some results provided by the proposed formulation were compared to numerical analyses shown by the literature. Second, the implemented finite element was evaluated using modal and frequency domain experiments for a cantilever aluminum beam with two piezoelectric transducers working under the d31 effects, and a free-free curved composite plate with four piezoelectric transducers working under the d33 effects. Comparisons between the natural frequencies and the frequency response functions amplitudes obtained from the experiments and the computational analyses were performed in order to discuss the limitations and potentialities of the proposed formulation.

1. Introduction

Since the development and commercial availability of manufactured piezoelectric fibers, such as the Macro Fiber Composite (MFC™ and Active Fiber Composite (AFC), several studies have been carried out on smart laminated composites, wherein some layers of a structural laminate composite are piezoelectric in nature [1]. These materials serve as sensors and/or actuators either attached or embedded in the structure. The piezoelectric behavior can be used in a variety of engineering applications, such as structure health monitoring, damage identification, vibration control and energy harvesting. However, one of the main challenges to design smart composites with embedded piezoelectric elements is the correct prediction and simulation of the mechanical, electrical and coupling behavior of the structure. The most common approach to simulate these structures is using commercial finite element analysis packages [1,2]. However, normally, that software offers only 3D-solid element and 2D-membrane elements with piezoelectric coupling capabilities. Hence, simulations of complex and/or large structures (e.g. wing or fuselage of aircrafts) become computationally impracticable, especially for thin structures and with high curvatures. In order to overcome these limitations, it is better to use shell elements like some researchers have proposed. In fact, it is possible to find at the literature different kinds of finite element formulations for laminated shell elements capable of including the piezoelectric effects [3]. One of the earliest works was written by Saravanos et al. [4] in which a layer-wise (LW) laminated plate element was adapted to include the piezoelectric effects by changing the lamina constitutive equations, considering the electrical–mechanical coupling. Similarly, several authors developed bi-dimensional finite elements with increasing advanced theories for the mechanical modeling, using either Equivalent Single-Layer (ESL) or layer-wise (LW) approaches. For example, regarding ESL approach, Zhen and Wanji [5] created an equivalent single layer plate element by using classic finite element theories. Zemčík et al. [6] developed a higher-order zigzag theory plate element. Neto et al. [7] made advances in the existing theories by studying the drilling degrees of freedom of the piezoelectric shells and frequency domain solutions. Following LW approach, for instance, Torres and Mendonça [8] investigated different higher order theories in piezoelectric composite elements by using a LW approach and the generalized Finite Element

* Corresponding author. Tel.: +55 16 3373 8612; fax: +55 16 3373 9590.
E-mail address: voltita@sc.usp.br (V. Tita).

http://dx.doi.org/10.1016/j.compstruct.2015.03.009
0263-8223/

Method (FEM). Nath and Kapuria [9] proposed a higher-order plate finite element with piezoelectric coupling. Regarding higher-order theories, several studies were carried out using Carrera's Unified Formulation (CUF) for composite materials [10], including piezoelectric coupling effects at the structural theory. Some researchers showed analytical closed form solutions for plates and cylinders via CUF with the inclusion of piezoelectric coupling effects [11–14]. In particular, Carrera et al. [15] compared several formulations using both ESL and LW theories, for both linear, higher-order and zigzag deformation theories, comparing the results to benchmark problems. However, finite elements based on higher-order theories are not computational effective, and some approaches for reduced models were proposed by Cinefra et al. [16], but this issue is still an open-problem.

In general, in terms of mechanical behavior predictions, the authors have discussed how the use of higher order theories does not lead to significant changes in the response of piezoelectric transducers when those are working under small strain conditions and recommend higher order theories only for accurate prediction of stresses or electrical displacements trough the thickness of the plate. By the other side, for the electrical behavior simulations, it is usually used a model with a uniform distribution of the electric field trough the thickness of the laminate. However, there are some works, which contribute to a better modeling of the electrical field, such as the formulation proposed by Sheikh et al. [17]. Those authors developed a laminated plate finite element with piezoelectric layers, considering zigzag theory for the electrical field. Marinković et al. [18] proposed a theory of linear variation for the electric field trough the thickness with d31 effects, considering an ESL approach for both mechanical and electrical domains. Yao and Lu [19] used higher order theories analogous to the higher order displacement theories. Besides, Carrera et al. [14,15] implemented several elements based on varying electrical field theories via CUF approach.

It is important to highlight that for the most of the works shown by the literature, the model of the electrical field is formulated using first or higher order theories and the continuous electrodes constitutive equations are written only for d31 effects. Moreover, in general, solutions for higher order theories or first order electrical field theories are shown only for plane elements, not for curved shells. These are strong limitations of the formulations in the case of simulating transducers with d33 effects positioned in curved structures, mainly on the outer layers of a laminate. For this situation, the bending and torsion efforts are the more significant contributions to the piezoelectric couplings, because they are more prevalent in curved structures [20]. Additionally, most of the proposed formulations have been compared to quasi-static analytical solutions and/or to other formulations. Indeed, there are few works, which shows a comparison between experimental data to numerical simulations by using new formulation. In some cases, the authors have performed quasi-static experiments. However, for piezoelectric transducers, this can be a problem due to both the unreliability of the sensing capacity in static loadings and the high potency required for actuation. Moreover, the most of applications of piezoelectric composite materials are related to dynamic loadings.

Regarding all aspects pointed above, the present work proposes a mathematical formulation of a shell finite element for piezoelectric smart composites. The formulation is based and extended upon the classic models for composite materials and contemplates piezoelectric transducers working not only under d31 effects, but also under d33 effects. This is done by elaborating a physically consistent first order electrical field polarization theory, which naturally fulfills the Gauss' law in strong form for both dielectric effects. It is important to notice that this formulation to treat d33 effects with a linear distribution of the electrical field over the thickness of a layer is a new scientific contribution. By the other side, the mechanical model is written by using the first order shear theory (FOST) and an implicit curvature formulation capable of simulating both single and double curved structures, as well as non-uniformly curved structures. As the objective of the current formulation is to simulate dynamic tests, where the requested results are displacement fields, there is no need for high accuracy in the stresses predictions. Besides, the computational efficiency is a very important issue in dynamic and frequency domain analyses due to the high number of steps in the simulation process. These facts justify the use of FOST over higher-order ones and an ESL approach for modeling the mechanical domain of the laminated shell, mainly if this proposed formulation will be used to simulate complex and/or large structures like aircraft wings and fuselages. However, as each smart layer behaves as an independent transducer, with its own electrical degrees of freedom, the electrical domain was modeled by using a LW approach. The proposed formulation was implemented considering an eight nodes serendipity quadratic finite element. Those shape functions were chosen due to constraints in the formulation regarding the differentiability of the mechanical strains. The element was implemented as a Fortran subroutine and compiled through the finite element commercial package Abaqus™ via User Element (UEL) subroutine [21]. First, results provided by proposed finite element were compared to numerical analyses shown by the literature. Second, the implemented finite element was evaluated by carrying out two dynamic experiments and comparing their results to numerical simulations. First, a cantilever aluminum beam with two piezoelectric transducers working under d31 effects was investigated, and then natural frequencies and experimental frequency response functions (FRFs) amplitudes were compared to the numerical ones. After that, a curved composite plate with four piezoelectric transducers working under d33 effects was investigated, and then natural frequencies and experimental frequency response functions (FRFs) amplitudes were compared to the computational analyses, as well. These comparisons show an additional important contribution of this article, because the limitations and potentialities of the proposed formulation are discussed based on experimental results carried out by own authors.

2. Mathematical model

The proposed mathematical formulation consist on an efficient smart shell finite element capable of simulating plane and curved laminated structures, which have one, some or all layers made of piezoelectric materials. The piezoelectric coupling model should be capable of simulating electrical boundary conditions, which allow for piezoelectric transducers working under both d33 and d13 effects. Thus, some hypotheses for the finite element model need to be considered. First, as the piezoelectric coupling for transducers working under the d33 effect is generated from the transversal shear strains $\gamma 13$, then these shear components cannot be neglected [24]. By itself, this fact means that the deformation theory must be at least a FOST. Furthermore, as these shear components are in the lamina level and as piezoelectric layers are thin, usually between 0.1 and 1.0 mm in thickness, the accuracy of the transversal shear distribution does not need to be high. In addition, the proposed formulation focuses on simulating dynamic test, where the displacement fields are the most important variables. Therefore, based on these arguments, higher order shear theories were not used. Lastly, due to the necessity of strain derivates for the first order electrical field theory, it is necessary to have at least a linear distribution of strains over the element, implying in a quadratic displacement field.