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An extension of the polar method to the First-order Shear Deformation Theory of laminates

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ABSTRACT

In this paper the Verchery's polar method is extended to the conceptual framework of the First-order Shear Deformation Theory (FSDT) of laminates. It will be proved that the number of independent tensor invariants characterising the laminate constitutive behaviour remains unchanged when passing from the context of the Classical Laminate Theory (CLT) to that of the FSDT. Moreover, it will also be shown that, depending on the considered formulation, the elastic symmetries of the laminate shear stiffness matrix depend upon those of membrane and bending stiffness matrices. As a consequence of these results a unified formulation for the problem of designing the laminate elastic symmetries in the context of the FSDT is proposed. The optimum solutions are found within the framework of the polar-genetic approach, since the objective function is written in terms of the laminate polar parameters, while a genetic algorithm is used as a numerical tool for the solution search. In order to support the theoretical results, and also to prove the effectiveness of the proposed approach, some novel and meaningful numerical examples are discussed in the paper.

1. Introduction

The problem of designing a composite structure is quite cumbersome and can be considered as a multi-scale design problem. The complexity of the design process is actually due to two intrinsic properties of composite materials, i.e. the heterogeneity and the anisotropy. Although the heterogeneity gets involved mainly at the micro-scale (i.e. the scale of constitutive "phases", namely fibres and matrix), conversely the anisotropy intervenes at both meso-scale (that of the constitutive lamina) and macro-scale (that of the laminate). It is well known that the material properties (and more generally the mechanical response) of an anisotropic continuum depend upon the direction. A consequence of anisotropy consists in the fact that the mechanical response of the material depends upon a considerable number of parameters (i.e. 21 for a general triclinic material, 13 for the monoclinic case, nine for the orthotropic one, five for the transverse isotropic case and two for an isotropic material).

Normally the Cartesian representation of tensors is employed to describe the behaviour of an anisotropic material in terms of Young's moduli, shear moduli, Poisson's ratios, Chentsov's ratios and mutual influence ratios, see [1]. While on one hand the Cartesian representation seems to be the "most natural" representation to describe the anisotropy, on the other hand it shows a major drawback: the above material parameters depend upon the coordinate system chosen for characterising the mechanical response of the continuum. As a consequence, the anisotropy of the material is described by a set of parameters which are not (tensor) invariant quantities and that represent the response of the material only in a particular frame and not in a general one.

Several alternative analytical representations can be found in literature. Some of them rely on the use of tensor invariants which allow for describing the mechanical behaviour of an anisotropic continuum through intrinsic material quantities. Of course, such representations do not imply a reduction in the number of parameters needed to fully characterise the material behaviour. Nevertheless, since these intrinsic material quantities are tensor invariants on one hand they allow to describe the mechanical response of the material regardless to the considered reference frame and on the other hand they let to better highlight some physical aspects that cannot be easily caught when using the Cartesian representation.

In the framework of the design of composite materials several analytical representations of (plane) anisotropy were developed in the past and among them the most commonly employed is that introduced by Tsai and Pagano [2]. In the context of this approach

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Notations

CLT Classical Laminate Theory
FSDT First-order Shear Deformation Theory
GA genetic algorithm
$\Gamma = \{O; x_1, x_2, x_3\}$ local (or material) frame of the elementary ply
$\Gamma^{\mathrm{l}} = \{O; x, y, z = x_3\}$ global frame of the laminate
θ rotation angle
$\{11, 22, 33, 32, 31, 21\} \Longleftrightarrow \{1, 2, 3, 4, 5, 6\}$ correspondence between tensor and Voigt's (matrix) notation for the indexes of tensors (local frame)
$\{xx, yy, zz, zy, zx, yx\} \Longleftrightarrow \{x, y, z, q, r, s\}$ correspondence between tensor and Voigt's (matrix) notation for the indexes of tensors (global frame)
Z_{ij}, $(i, j = 1, 2$ or $i, j = x, y)$ second-rank plane tensor using tensor notation (local and global frame)
L_{ijkl}, $(i, j, k, l = 1, 2$ or $i, j, k, l = x, y)$ fourth-rank plane tensor using tensor notation (local and global frame)
U_i $(i = 1, \ldots, 7)$ parameters of Tsai and Pagano
[Q] 3×3 in-plane reduced stiffness matrix of the constitutive lamina (Voigt's notation)
$[\widehat{\mathrm{Q}}]$ 2×2 out-of-plane reduced stiffness matrix of the constitutive lamina (Voigt's notation)
$T_0, T_1, R_0, R_1, \Phi_0, \Phi_1$ polar parameters of a fourth-rank plane tensor (also used for the lamina in-plane reduced stiffness matrix [Q])
T, R, Φ polar parameters of a second-rank plane tensor (also used for the lamina out-of-plane reduced stiffness matrix $[\widehat{\mathrm{Q}}]$)
{N} 3×1 vector of membrane forces (per unit length), Voigt's notation
{M} 3×1 vector of bending moments (per unit length), Voigt's notation
{F} 2×1 vector of shear forces (per unit length), Voigt's notation
$\{\varepsilon_0\}$ 3×1 vector of in-plane strains of the laminate middle plane, Voigt's notation
$\{\chi_0\}$ 3×1 vector of curvatures of the laminate middle plane, Voigt's notation
$\{\gamma_0\}$ 2×1 vector of the out-of-plane shear strains of the laminate middle plane, Voigt's notation
[A], [B], [D] 3×3 matrices of laminate membrane, membrane/bending coupling and bending stiffness, respectively (Voigt's notation)
$[\mathrm{A}^*], [\mathrm{B}^*], [\mathrm{D}^*]$ 3×3 matrices of laminate homogenised membrane, membrane/bending coupling and bending stiffness, respectively (Voigt's notation)
[H] 2×2 matrix of laminate out-of-plane shear stiffness, (Voigt's notation)
$[\mathrm{H}^*]$ 2×2 matrix of laminate homogenised out-of-plane shear stiffness, (Voigt's notation)
$[\mathrm{C}^*]$ 3×3 laminate homogeneity matrix
$T_{0A^*}, T_{1A^*}, R_{0A^*}, R_{1A^*}, \Phi_{0A^*}, \Phi_{1A^*}$ polar parameters of $[\mathrm{A}^*]$
$T_{0B^*}, T_{1B^*}, R_{0B^*}, R_{1B^*}, \Phi_{0B^*}, \Phi_{1B^*}$ polar parameters of $[\mathrm{B}^*]$
$T_{0D^*}, T_{1D^*}, R_{0D^*}, R_{1D^*}, \Phi_{0D^*}, \Phi_{1D^*}$ polar parameters of $[\mathrm{D}^*]$
$T_{H^*}, R_{H^*}, \Phi_{H^*}$ polar parameters of $[\mathrm{H}^*]$
E_i, $(i = 1, 2, 3)$ Young's moduli of the constitutive lamina (material frame)
G_{ij}, $(i, j = 1, 2, 3)$ shear moduli of the constitutive lamina (material frame)
ν_{ij}, $(i, j = 1, 2, 3)$ Poisson's ratios of the constitutive lamina (material frame)
t_{ply} thickness of the constitutive lamina
n number of layers
$\{\delta_k\}$ $(k = 1, \ldots, n)$ vector of the layers orientation angles
h overall thickness of the laminate
Ψ overall objective function for the problem of designing the elastic symmetries of the laminate
{f} 21×1 vector of partial objective functions
[W] 21×21 positive semi-definite diagonal weight matrix
$\widehat{R_{0A^*}}, \widehat{R_{1A^*}}, \widehat{\Phi_{0A^*}}, \widehat{\Phi_{1A^*}}$ imposed values for the polar parameters of matrix $[\mathrm{A}^*]$
$\widehat{R_{0D^*}}, \widehat{R_{1D^*}}, \widehat{\Phi_{0D^*}}, \widehat{\Phi_{1D^*}}$ imposed values for the polar parameters of matrix $[\mathrm{D}^*]$
N_{pop} number of populations
N_{ind} number of individuals
N_{gen} maximum number of generations
p_{cross} crossover probability
p_{mut} mutation probability

they introduce seven parameters $U_i, (i = 1, \ldots, 7)$ which are expressed in terms of the six independent Cartesian components of a fourth-rank elasticity-like plane tensor (i.e. a tensor having both major and minor symmetries) written in the local frame $\Gamma = \{O; x_1, x_2, x_3\}$:

$$\begin{aligned}
U_1 &= \frac{3L_{1111} + 2L_{1122} + 3L_{2222} + 4L_{1212}}{8},\\
U_2 &= \frac{L_{1111} - L_{2222}}{2},\\
U_3 &= \frac{L_{1111} - 2L_{1122} + L_{2222} - 4L_{1212}}{8},\\
U_4 &= \frac{L_{1111} + 6L_{1122} + L_{2222} - 4L_{1212}}{8},\\
U_5 &= \frac{L_{1111} - 2L_{1122} + L_{2222} + 4L_{1212}}{8},\\
U_6 &= \frac{L_{1112} + L_{1222}}{2},\\
U_7 &= \frac{L_{1112} - L_{1222}}{2}.
\end{aligned} \tag{1}$$

The main drawbacks of this representation are basically three: firstly not all parameters U_i are tensor invariants, secondly they do not have a simple and immediate physical meaning and, finally, they are not all independent. Indeed, U_5 can be expressed in terms of U_1 and U_4 as:

$$U_5 = \frac{(U_1 - U_4)}{2}. \tag{2}$$

In 1979 Verchery [3] introduced the polar method for representing fourth-rank elasticity-like plane tensors. This representation has been enriched and deeply studied later by Vannucci and his co-workers [4–8]. The polar method relies upon a complex variable transformation by taking inspiration from a classical technique often employed in analytical mechanics, see for instance the works of Kolosov [9] and Green and Zerna [10]. As it will be briefly described in Section 2, the main advantages of the polar formalism are at least three: (a) it is a representation of anisotropy which is based on tensor invariants, (b) such invariants have an immediate physical meaning which is linked to the different (elastic) symmetries of the tensor and (c) the change of reference frame can be expressed in a straightforward way.

Concerning the problem of the design of a composite structure, the polar method has been applied, up to now, only in the framework of the Classical Laminate Theory (CLT) for different real-life engineering applications, see [11–17]. Nevertheless, the results obtained by using the polar method in the context of the CLT are not sufficiently accurate for those applications involving