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Exact refined buckling solutions for laminated plates under uniaxial and biaxial loads

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ABSTRACT

This paper presents a unified Lévy-type solution procedure for the buckling analysis of both thin and thick composite plates under biaxial loads. The plates are simply-supported at two opposite edges, while the two remaining sides are subjected to any combination of simply-supported, clamped and free conditions. The problem is formulated in the context of a variable-kinematic approach, offering the advantage of automatically handling theories of various order. Both layerwise and equivalent single layer theories are considered. The governing equilibrium equations are derived analytically from the Principle of Virtual Displacements (PVD), and are solved exactly referring to the Lévy-type procedure. The accuracy of the predictions is demonstrated by comparison with results available in literature, including exact 3D solutions. A comprehensive set of benchmark results is provided for plates subjected to different loading and boundary conditions and characterized by various width-to-thickness ratios.

1. Introduction

Composite structures are widely used in aerospace, civil and marine applications. They are often required to operate under loading conditions that can promote elastic instability, such as in the case of uniaxial or biaxial compression. During the design phase, buckling loads can be predicted with various approaches, including numerical, analytical and semi-analytical techniques. Often, ad hoc tools are developed to improve the design process, making necessary the availability of reference solutions to check the accuracy of the results and validate new methods.

In general, exact buckling solutions can be derived for a limited set of stacking sequences, loading and boundary conditions. The Navier method can be applied to study cross-ply rectangular plates, simply-supported at the four edges, and subjected to biaxial compression. The method leads to a standard eigenvalue problem that can be solved in closed-form or numerically, depending on the underlying plate theory. Another approach is the Lévy method, which is suitable for rectangular plates with two parallel edges simply-supported, and any combination of free, simply-supported and clamped conditions at the two remaining edges. In this case, the buckling problem is reduced to the solution of a transcendental equation, whose solution is sought numerically. Other boundary and loading conditions can be studied referring to approximate solution procedures, the most common being the Ritz, Galerkin, and modified Galerkin methods.

The application of the Navier and Lévy methods to the analysis of thin composite plates is discussed in Ref. [1], where classical plate theory (CPT) is adopted. The application of approximate solution strategies to plates with various boundary conditions, including elastically restrained edges, is found in Ref. [2–4]. Thin plate solutions are a useful reference during the analysis of plates with relatively high values of width-to-thickness ratio. However, they are inadequate when moderately thick plates are of concern, and the contribution of transverse shear deformation is not negligible.

First-order shear deformation theory (FSDT) is the simplest approach to account for transverse shear deformations. It has been applied to the buckling analysis of moderately thick plates in several studies. For instance, exact closed-form solutions are derived by Liew and co-workers [5] for the buckling analysis of simply-supported composite plates under biaxial loads using the Navier method. Exact solutions have been derived also for isotropic [6] and cross-ply [1,7,8] plates under various boundary conditions, referring to the Lévy-type approach. One drawback related to FSDT is the need for shear correction coefficients, as the underlying kinematic assumptions imply constant transverse shear deformation.

The introduction of correction coefficients can be avoided by employing a mixed first-order deformation theory, as discussed by Zenkour [9]. In his work, the Galerkin method is applied and

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approximate buckling loads are derived for any combination of boundary conditions at the four edges.

Higher-order theories are an effective approach to deal with relatively thick plates and avoid the need for shear correction factors. The basic idea is to represent the displacement field along the thickness with a polynomial expansion. In most cases, the in-plane displacements are represented with a cubic expansion, while the out-of-plane displacement is assumed to be quadratic at most. Exact buckling solutions are obtained by Reddy and Phan [10] for simply-supported plates applying the Navier method, while Hadian and Nayfeh [11] derive Lévy-type solutions for cross-ply laminates, using the state-space approach together with the method of orthonormalization.

Another application of the Lévy-type approach to high-order theories is found in the works of Khdeir [12–14] where biaxial buckling of symmetric and unsymmetric laminated plates is investigated for different boundary conditions. However, boundary conditions do not properly account for the effect of in-plane loads, as observed in Refs. [6,8].

Shufrin and Eisenberger [15] present a procedure based on the extended Kantorovich method referring to first-order and Reddy's high-order deformation theories. Buckling loads are derived for different loading and boundary conditions. In the case of two parallel simply-supported edges and biaxial loading condition, the solutions are exact.

Numerical solutions using collocation with radial basis functions and third-order shear deformation theory are presented by Ferreira et al. [16] for the uniaxial and biaxial buckling of laminated plates.

Isotropic plates, simply-supported along the four edges and subjected by pure compression, are studied by Matsunaga [17,18] in the context of a two-dimensional higher-order theory, based on the method of power series expansion. The approach is particularly suited for the study of very thick plates.

Recently, a two variable refined theory [19] has been proposed to guarantee a quadratic variation of the transverse shear strain along the thickness, thus avoiding the use of a shear correction factor as required in FSDT theory. The theory enforces the satisfaction of the null traction boundary condition at the top and the bottom of the panel, and offers the advantage of leading to governing equations similar to those obtained in the context of CPT. The application of the two variable refined theory is discussed by Kim et al. [20] and Thai and Kim [21] with regard to the biaxial buckling of both isotropic and orthotropic plates. Exact solutions are derived using the Navier and Lévy methods, respectively.

A relatively limited amount of works in the literature regards the derivation of exact buckling solutions in the context of 3D elasticity theory and, in any case, they are restricted to the boundary conditions of simple-support at the four edges. An example is provided by the works of Srinivas and Rao [22], where buckling solutions are reported for cross-ply plates under pure and biaxial compression. Cross-ply plates are studied also by Noor [23], where 3D governing equations are solved using a high-order finite difference scheme. Solutions to the 3D problem are reported in a closed-form manner by Wittrick [24] for isotropic plates loaded in compression. More recently, a 3D approach has been proposed by Gu and Chattopadhyay [25] where buckling of laminated plates is reduced to the solution of a nonlinear eigenvalue problem.

The main restriction to the use of 3D theory is the time needed to derive the solutions, which can be not adequate in the context of sensitivity or parametric studies.

A powerful approach to automatically consider several plate theories within the same theoretical framework is the unified formulation proposed by Carrera [26,27], often referred to as Carrera unified formulation (CUF). This technique relies on hierarchically-ordered approximations to describe the displacement field in the thickness direction, and allows to range from classical 2D to quasi-3D layerwise models. It follows that low-order theories can be used when thin plates are of concern, while high-order approximations can be adopted to study thicker plates. Concerning buckling problems, variable-kinematic theories have been applied by D'Ottavio and Carrera [28] to study the instability of plates and shells under biaxial loads. Exact solutions are derived with the Navier procedure, and are restricted to boundary conditions of simple-support at the four edges. An extension to the buckling of anisotropic plates under various kind of loading and boundary conditions is provided by Fazzolari and Carrera [29] and Nali and Carrera [30]. In both cases, the solutions are not exact, as they are derived from the approximate solution of the governing equations by means of the Ritz, Galerkin, generalized Galerkin methods [29] and the finite element method [30].

To the best of the authors' knowledge, no exact solutions are available in the literature for the biaxial buckling of flat plates under various boundary condition, based on the variable-kinematic theory. A formulation is here presented to obtain the exact buckling solutions for plates with two parallel edges simply-supported, and the two remaining edges subjected to any combination of clamped, free and simply-supported edges. Governing equations are derived from the Principle of Virtual Displacements (PVD), which is formulated in a fully nondimensional manner, and are successively solved referring to the Lévy-type procedure in conjunction with the state-space approach. The strain–displacement relation is modeled by using the Green–Lagrange expression, with and without the von Kármán approximation. The results are compared with those available in the literature, including exact three-dimensional solutions, revealing excellent accuracy. A number of results is finally reported for different boundary conditions and loading conditions to be used in the future for benchmarking purposes.

2. Plate description

A multilayered rectangular plate is considered, as illustrated in Fig. 1. The plate is obtained by the stacking of an arbitrary number N_l of orthotropic layers, each of them characterized by a thickness h_k and subjected to a three-dimensional state of stress. It is assumed that the layers are perfectly bonded along the common surfaces, so that interlaminar compatibility of the displacements is implied. The plate has length a, width b and total thickness equal to h.

In the present formulation, two classes of theories of order N are considered, namely the equivalent single-layer displacement-based (EDN) and the layerwise displacement-based (LDN) theories [27]. In the former, one single reference system is taken on the midsurface of the laminate, as sketched in Fig. 2(a); in the latter, a number of N_l reference systems are taken on the midsurfaces of each single ply, as illustrated in Fig. 2(b). In both cases, the z-axis

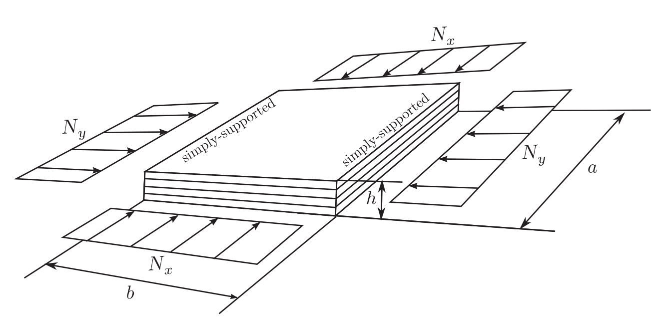

Fig. 1. Multilayered plate subjected to biaxial loading.