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Improved one-dimensional model of piezoelectric laminates for energy harvesters including three dimensional effects

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ABSTRACT

The application of piezoelectric composites in energy harvesters is continuously increasing even at the microscale, with the immediate corollary of a fundamental need for improved computational tools for optimization of performances at the design level. In this paper, a refined, yet simple model is proposed with the aim of providing fast and insightful solutions to the multi-physics problem of energy harvesting via piezoelectric layered structures. The main objective is to retain a simple structural model (Euler–Bernoulli beam), with the inclusion of effects connected to the actual three-dimensional shape of the device. A thorough presentation of the analytical model is presented, along with its validation by comparison with the results of fully 3D computations.

1. Introduction

Piezoelectric materials have been widely used in many technological applications at the macroscopic scale. In recent times, such materials have been entering in the field of Micro-Electro-Mechanical Systems (MEMS): in particular, the coupling between electrical and mechanical behavior is feasibly used in MEMS energy harvesters (which exploit the so-called "direct effect") and actuators (based on the "indirect effect"). Piezoelectric MEMS have been proven to be an attractive technology for harvesting small magnitudes of energy from ambient vibrations. This technology promises to eliminate the need for batteries or complex wiring in microsensors/microsystems, moving closer towards battery-less, autonomous sensors systems and networks which recovered on-site the energy they need to fulfill their tasks. At the present time, most of the piezoelectric harvesters reported in the literature are cantilever laminated beams and plates with thin films of lead zirconate titanate Pb(Zr,Ti) O_3 (PZT) on Si or SiNx substrate [1–3]. The multi-physics simulation of piezoelectric effects can be obtained by considering that the structural members are represented by a laminate composite with piezoelectric and silicon layers [4], the active layer is then attached to an external load resistance which reproduces the circuitry employed for the power management. Numerous models have been reported, but the effect of 3D strain field on the coupling has been considered only in few cases [5,6]. In this paper, a simple 1D model is built in order to simulate piezoelectric thin beams and plate harvesters [7]. Starting from the fully coupled 3D constitutive equations of piezoelectricity, appropriate hypotheses are introduced to model strains and stresses so that the 1D model takes into account the 3D effects. It is worth noting that such behavior is usually neglected if the standard mechanical response of the beam is considered, in view of a small difference with respect to analytical results. Conversely, in the presence of piezoelectric coupling, the effects connected to the actual aspect ratio of the cantilever involve a significant variation of the results in terms of electrical quantities. Consequently, the problem should be carefully studied in order to provide a reliable estimate of the energy production in a wide range of geometrical configurations. The sectional behavior of the beam is studied through the Classical Lamination Theory (*CLT*) specifically modified in order to introduce the piezoelectric coupling and a reduced order model is built through separation of time and space variables and the introduction of a suitable shape function for the beam deformation. The resulting coupled equations are solved analytically in the frequency domain and numerically in the time domain by means of step-by-step integration (α-method). For the sake of comparison, simulations have been carried out also by means of a fully 3D model implemented in a commercial code, coupled within an *ad hoc* created procedure to introduce the effect of the electrical circuit. The results show that the proposed model is in excellent agreement with the numerical outcomes, with substantial improvements with respect to the standard analytical solutions. The new constitutive equations can be employed for simulating the performances of

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innovative piezoelectric energy harvesters, including nonlinear [8–10], bistable and frequency-up-converter harvesters [11].

2. One dimensional model with three-dimensional effects

The piezolaminated cantilever beam is presented in Fig. 1, where L is the length, h the total thickness and b the width. The x_3-coordinate originates in the neutral axis and is directed downwards, x_1-coordinate lies along the beam axis while the x_2-coordinate originates in the middle of the beam, so that $-b/2 \leqslant x_2 \leqslant b/2$. A piezoelectric layer (for instance, made by PZT material) is placed on top of the beam substrate and is activated in d_{31}-mode when the beam deflects: this means that the axial deformation of the layer causes an electric field in the vertical direction (along x_3-axis).

To implement d_{31}-mode, the polarization vector is opposite to x_3-axis and the electrodes span both the upper and the lower surfaces of PZT thin film. In this way, when the PZT layer is stretched along x_1-axis, the generated charge is collected by the electrodes. The bottom electrode is grounded while the other is attached to an external load resistance (R) which represents an ideal external circuit employed for the management of the power generated by PZT. Even though it is not developed here, the model described in the following can be easily extended to d_{33}-mode piezoelectric harvesters. The standard notation of piezoelectric analysis is adopted: stress components are denoted by the capital letter T and strain components by the capital letter S. Voigts notation is employed herein. T_{ij}, S_{ij}, E_i, D_i are stress, strain, electric field and electric displacement field components; E is the Young's modulus and ν is the Poissons coefficient, e_{ij} and ε^S_{ii} are the piezoelectric stress constant and the dielectric constant computed at constant mechanical stress. It is worth noting that, in view of the geometric features of the problem, only a subset of the tensor and vector components should be explicitly involved in the analysis. The beam final stack is not homogeneous since different deposited layers are employed. The mechanical response of the layered beam can be obtained by means of a number of theories widely studied by a group at Politecnico di Torino, examined in [12]. The same group proposed a refined electro-mechanical beam formulation which uses Lagrange-type polynomials to interpolate the unknowns over the beam cross section. In such way linear, bi-linear, and quadratic approximations of the displacement and electric field over the beam cross-section are obtained [13,14]. The electromechanical model has been used in conjunction with Carrera Unified Formulation (CUF) [15] which allows one to introduce any order expansions of the displacement unknowns over the beam section. Herein, much simpler models are used to describe the beam-cross section behavior. These assumptions limit the validity of the model to extremely thin plates and to simple shapes of the laminated structures since these hypotheses apply to micro-fabricated laminated structures. Herein, the cross-sections orthogonal to x_1 axis are considered to remain plane after deformation and the displacement vector results:

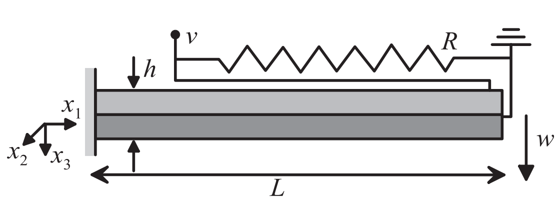

Fig. 1. Cantilever piezoelectric harvester, the piezoelectric material (light grey) is placed on a structural layer (dark grey).

$$\mathbf{s} = \begin{bmatrix} -x_3 w_3'(x_1;t) \\ \hat{s}_2(x_1,x_2,x_3,\Lambda;t) \\ y(t) + w_3(x_1;t) + \hat{s}_3(x_1,x_2,x_3,\Lambda;t) \end{bmatrix} \tag{1}$$

$y(t)$ denotes an external motion of the reference along x_3 axis while w_3 is the average vertical displacement of the beam cross-section ($\bullet'$ means derivative of $\bullet$ with respect to the variable). The kinematic model presented in Eq. (1) is an improvement of Euler–Bernoulli model which is justified as long as the piezoelectric beam is considered thin and the layer elastic properties are of the same order. The modification accounts for additional vertical and in-plane displacements which depend on x_2 and x_3 coordinates. In such a way, if $\hat{s}_2$ and $\hat{s}_3$ are appropriately defined, it is possible to build three dimensional stress–strain states. These displacements are also assumed to depend on a parameter which is a sort of *in-plane slenderness* of the beam defined as $\Lambda = L/b$. Functions $\hat{s}_2$ and $\hat{s}_3$ must satisfy some specific hypotheses in order to be compatible to physical stress–strain states in the beam. First, since the beam is considered thin, the in-the-thickness stress must be null, $T_{33} = 0$. Second, the in-plane stress must be $T_{22} = 0$ at $x_2 = \pm b/2$. Moreover, when $\Lambda \to 0$ the beam is infinitely wide and the strain condition $S_{22} = 0$ must always be verified; on the other hand when $\Lambda \to \infty$ the beam width is null and $T_{22} = 0$ has to be guaranteed. Considering piezoelectrics, the constitutive equations describing these two limit states read [5]:

- Null Transverse Deformation (*NTD*), $\Lambda \to 0 : T_{33} = 0$ and $S_{22} = 0$

$$T_{11} = \frac{E}{1-\nu^2} S_{11} - \left(e_{31} - \frac{\nu}{1-\nu} e_{33}\right) E_3 \tag{2a}$$

$$T_{22} = \frac{E\nu}{1-\nu^2} S_{11} - \left(e_{32} - \frac{\nu}{1-\nu} e_{33}\right) E_3 \tag{2b}$$

$$S_{33} = \frac{\nu}{1-\nu^2} S_{11} - \frac{(1-2\nu)(1+\nu)}{E(1-\nu)} e_{33} E_3 \tag{2c}$$

$$D_3 = \left(e_{31} - \frac{\nu}{1-\nu} e_{33}\right) S_{11} + \left(\varepsilon^S_{33} + \frac{(1-2\nu)(1+\nu)}{E(1-\nu)} e^2_{33}\right) E_3 \tag{2d}$$

- Null Transverse Stress (*NTS*), $\Lambda \to \infty$ or $x_2 = \pm b/2 : T_{33} = 0$ and $T_{22} = 0$

$$T_{11} = E S_{11} - (e_{31} - \nu e_{32} - \nu e_{33}) E_3 \tag{3a}$$

$$S_{22} = -\nu S_{11} + \frac{1-\nu^2}{E}\left(e_{32} - \frac{\nu}{1-\nu} e_{33}\right) E_3 \tag{3b}$$

$$S_{33} = -\nu S_{11} + \frac{1-\nu^2}{E}\left(-\frac{\nu}{1-\nu} e_{32} + e_{33}\right) E_3 \tag{3c}$$

$$D_3 = (e_{31} - e_{32}\nu - e_{33}\nu) S_{11} + \left(\frac{1-\nu^2}{E}\left((e_{32}+e_{33})^2 - \frac{2e_{32}e_{33}}{1-\nu}\right) + \varepsilon^S_{33}\right) E_3 \tag{3d}$$

Moreover, in-plane electric displacement components are null accordingly to the position of electrodes.

Finally, the kinematic state given by Eq. (1) must be built such that the mean shear deformations are nullified. This is guaranteed if $\hat{s}_2$ is an odd function of x_2 and neglecting all quadratic terms in x_3 (since the thickness is supposed to be small compared to other dimensions).

By imposing $T_{33} = 0$, which holds in all cases, one finds a mandatory correlation that links the in-the-thickness to the in-plane strains:

$$S_{33} = -\frac{\nu}{1-\nu}(S_{11} + S_{22}) + c_3 \tag{4}$$