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Static and frequency optimization of folded laminated composite plates using an adjusted Differential Evolution algorithm and a smoothed triangular plate element

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ABSTRACT

The paper proposed a coupled numerical method for the static and fundamental frequency optimization of folded laminated composite plates. In the optimization schemes, the objective function is to minimize the strain energy for static problems and to maximize the fundamental frequency for free vibration problems. The fiber orientations are taken as design variables which are discrete integer values between -90° and 90°. For analyzing effectively the behavior of folded laminated composite plates, a recently proposed smoothed triangular plate element, named the cell-based smoothed discrete shear gap method (CS-DSG3), is applied. For searching the optimal solution which contains discrete integer values, an adjusted Differential Evolution (aDE) algorithm is proposed by integrating the conventional Differential Evolution (DE) for searching the optimal continuous solution with a novel technique for handling discrete integer variables and a mutation strategy. The reliability and effectiveness of the proposed aDE are verified by comparing its numerical results with those of other algorithms in literature such as Genetic Algorithm (GA), Particle Swam Optimization (PSO), etc.

1. Introduction

Nowadays, folded plate structures such as roofs, cooling towers, sandwich plate cores, vehicle chassis, ship hulls and many other structures are used commonly in various engineering disciplines. This is because folded plate structures possess many advantages such as lightweight, easy manufacture, suitable cost, and high load carrying capacity compared to flat plates. Due to such superior characteristics and wide range of application, the analyses of folded plate structures have attracted a certain attention from researchers around the world [1–12]. Goldberg and Leve [1] are regarded as the first persons to give the exact static solution of folded plates. Yitzhaki and Reiss [7] took the moments along the joints of folded plates as unknowns, and applied the slope deflection method in the analysis of the folded plates. Based on Vlasov's theory of thin-wall beams, Yoseph et al. [8] proposed an approximated method for folded plates in 1989. Peng et al. [11] used Meshfree Method in association with First-order Shear Deformation Theory (FSDT) to analyze the bending behavior of folded plates in 2006.

Composite materials, which possess many superior characteristics, have been used widely for folded plate structures. Following the trend, many numerical techniques for analysis of folded laminated composite plates have been proposed [13–17]. Guha Niyogi et al. [13] used a nine-node plate element that incorporates first-order transverse shear deformation and rotary inertia to predict the free and forced vibration response of folded laminated composite plate structures. In 2005, Haldar and Sheikh [14] used a shear flexible element to analyze the free vibration of both isotropic and composite plates. In 2011, Peng et al. [17] proposed a mesh-free method based on the first-order shear deformation theory (FSDT) for bending analysis of folded laminated plates. However in comparison, it can be seen that studies in literature related to the analysis of folded composite plates using three-node triangular Mindlin plate elements are somewhat still limited. This paper hence partly aims to contribute a new numerical procedure for the analyses of folded composite plates by further extending a

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new smoothed triangular Mindlin plate element, the cell-based smoothed discrete shear gap method (CS-DSG3), proposed recently by Nguyen-Thoi et al. [18].

In terms of developing optimization algorithms for solving the optimization problems of folded plate structures, the studies so far are still very limited and usually focus on local optimization methods. For example, Hinton et al. [19] used Sequential Quadratic Programming (SQP) to find the minimum strain energy for folded plate. In this work, the authors used two-node finite strips method with a constraint of constant structural volume. Ozakca et al. [20,21] contributed a shape and buckling optimization for prismatic folded plates. Topal and Uzman [22] solved the optimization problem for folded laminated composite plates with only one design variable by using the Adjusted Feasible Direction (MFD) method. Recently, Ghasemi et al. [23–25] proposed and solved optimization problems for fiber reinforced composite structures in association with uncertainties using a nested double loop Reliability Based Design Optimization (RBDO) method including Optimality Criteria (OC) method, Reliability Index (RI) method and Non-Uniform Rational B-spline (NURBS). However, optimization methods used in these work possess two main drawbacks related to local optimization methods. Firstly, they depend too much on the initial point provided by users. As a result, if the initial point is not chosen well, especially for the optimization problems with many design variables, it is very hard or even impossible for local search methods to find the optimum solution. Secondly, since local search methods use gradient information for searching the solution, the solution obtained by these methods is easily trapped in local optimal solutions if the problem has more than one local extreme. For the optimization problems of folded laminated composite plates, the relation between design variables (i.e. laminates ply angles) and fitness functions (i.e. strain energies or normalized frequencies) is highly nonlinear with a lot of local extremes. As a result, it would be very hard to choose an appropriate initial point leading to global optimum solution for each problem. Therefore, it is necessary to further develop the global optimization methods for solving the optimization problems of folded laminated composite plate structures.

This paper hence aims to fill this gap by proposing a novel global optimization scheme for solving the optimization problems of folded laminated composite plates. In this scheme of optimization, an adjusted Differential Evolution (aDE) for finding the global optimal solution is combined with the CS-DSG3 for analyzing the behavior of folded laminated composite plates. The proposed aDE is a modified version of the global optimization algorithm Differential Evolution [26–28], which is inspired from the evolution of nature. In the optimization scheme, the objective functions are considered as minimization of elastic strain energy for static and maximization of fundamental frequency for free vibration problems. The design variables are fiber orientations of layers in the folded composite plates. Because fiber orientation values are designed as integer values, a novel technique for handling integer variables and a mutation strategy are proposed to make the conventional DE become more suitable and efficient for searching the optimal integer solution of the present optimization problem. The reliability and effectiveness of the proposed aDE will be verified by comparing its numerical results with those of other algorithms in literature such as Genetic Algorithm (GA), Particle Swam Optimization (PSO), etc.

The paper is then organized as follows. Section 2 generally introduces the formulation of the CS-DSG3. In section 3, the conventional DE and the novel aDE are presented. Section 4 performs numerical examples to verify the reliability and efficiency of the proposed method. Finally, some conclusions are withdrawn in section 5.

2. A brief on finite element formulation for folded laminated composite plates using a smoothed triangular plate element

2.1. Weakform for the analysis of the folded laminated composite plates

In this paper, folded laminated composite plates in the global coordinate system *OXYZ* are modeled by the flat laminated composite shell elements in local coordinate system *oxyz*. The flat laminated composite shell element is a combination of a membrane element and a laminated composite plate element in which the middle surface of plate is chosen as the reference plane that occupies a domain $\Omega \in \mathbb{R}^3$.

Let u, v, w be the displacements of the middle plane in the x, y, z directions, and $\beta_x, \beta_y, \beta_z$ be the rotations of the middle plane around y-axis, x-axis, and z-axis respectively. Then, the unknown vector of a folded laminated composite plate includes six independent field variables, $\mathbf{u} = \{u \quad v \quad w \quad \beta_x \quad \beta_y \quad \beta_z\}^{\mathrm{T}}$, at any point in the problem domain in local coordinate system *oxyz*.

According to the FSDT theory, the membrane strain $\boldsymbol{\varepsilon}_m$, the curvature of the shell $\boldsymbol{\kappa}$ and the shear strains $\boldsymbol{\gamma}$ are respectively defined as

$$\boldsymbol{\varepsilon}_m^{\mathrm{T}} = \left\{ \frac{\partial u}{\partial x} \; \frac{\partial v}{\partial y} \; \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\}; \quad \boldsymbol{\kappa}^{\mathrm{T}} = \left\{ \frac{\partial \beta_x}{\partial x} \; \frac{\partial \beta_y}{\partial y} \; \frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} \right\};$$
$$\boldsymbol{\gamma}^{\mathrm{T}} = \left\{ \frac{\partial w}{\partial x} + \beta_x \; \frac{\partial w}{\partial y} + \beta_y \right\} \tag{1}$$

The Galerkin weakform of the folded laminated composite plate can now be written as

$$\int_\Omega \left\{ \delta(\boldsymbol{\varepsilon}_m)^T \quad \delta(\boldsymbol{\kappa})^T \quad \delta(\boldsymbol{\gamma})^T \right\} \begin{bmatrix} \mathbf{D}_m & \mathbf{D}_{mb} & 0 \\ \mathbf{D}_{mb} & \mathbf{D}_b & 0 \\ 0 & 0 & \mathbf{D}_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_m \\ \boldsymbol{\kappa} \\ \boldsymbol{\gamma} \end{Bmatrix} d\Omega$$
$$+ \int_\Omega \delta(\mathbf{u})^T \mathbf{m}\ddot{\mathbf{u}} d\Omega = \int_\Omega \delta\mathbf{u}^{\mathrm{T}} \mathbf{b} d\Omega \tag{2}$$

where $\mathbf{b}$ is the distributed load applied on the plate; $\mathbf{m}$ is mass matrix containing the mass density of the material ρ, and expressed by

$$\mathbf{m} = \sum_{k=1}^{L} \rho_{(k)} \int_{z_k}^{z_{k+1}} \begin{bmatrix} 1 & 0 & 0 & z & 0 & 0 \\ 0 & 1 & 0 & 0 & z & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ z & 0 & 0 & z^2 & 0 & 0 \\ 0 & z & 0 & 0 & z^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dz \tag{3}$$

In Eq. (2), $\mathbf{D}_m, \mathbf{D}_{mb}, \mathbf{D}_b$ and $\mathbf{D}_s$ are, respectively, the material matrices related to the membrane, the coupling membrane and bending, the bending and the shear deformations, and calculated ass

$$(\mathbf{D}_m, \mathbf{D}_{mb}, \mathbf{D}_b) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \left(\overline{Q}_{ij}\right)_k (1, z, z^2) dz \quad i,j = 1,2,6,$$
$$(\mathbf{D}_s) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \kappa \left(\overline{Q}_{ij}\right)_k dz \quad i,j = 4,5, \tag{4}$$

where $\kappa = 5/6$ is the shear correction coefficient; $\overline{Q}_{ij} = \mathbf{T}^T Q_{ij} \mathbf{T}$ are the transformed material matrices of the kth layer of the plate; Q_{ij} are calculated using material constants $E_1, E_2, G_{12}, G_{13}, G_{23}, v_1, v_2$; $\mathbf{T}$ is the transformation matrix which transforms material properties from standard direction 0° to an arbitrary value of fiber