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A series solution for the vibrations of composite laminated deep curved beams with general boundaries

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ABSTRACT

In this paper, a series solution is derived for the vibration analysis of composite laminated deep curved beams with general boundary conditions. The effects of shear deformation, inertia rotary and deepness term are considered in the formulation. Under the current framework, the governing equations and the related boundary equations are obtained via the Hamilton's principle. And each of beam displacements, regardless of boundary conditions, is expanded as a modified Fourier series composed of a standard cosine Fourier series and certain supplementary terms introduced to remove the potential discontinuities at the ends, thus ensure and accelerate the convergence of the series representation. The characteristic equations are then derived directly in an exact sense by solving the equations of motion in matrix form by combining the associated boundary equations and the modified Fourier series representation. The convergence and accuracy of the solution are tested and validated by several numerical cases against the results available in the literature, with excellent agreements obtained. A systematic parametric study is also performed regarding the effects of shear deformation and inertia rotary, deepness term, boundary conditions, lamination schemes, material and geometrical parameters. Finally, several numerical results of composite laminated deep and shallow curved beams with different geometry dimensions are presented for various boundary conditions and lamination schemes, which may serve as benchmark solutions for the future researches in this field.

1. Introduction

In practical applications ranging from outer space to the deep ocean, engineering structures, such as aircrafts, rockets, automobiles, vessels and submarines, are often applied in complex environment conditions and can be subjected to various dynamic loads which can lead to the vibratory behaviors of the structures. In all these applications, the engineering structures may fail and collapse because of material fatigue resulting from vibrations. Therefore, it is of particular importance to understand the structural vibrations and reduce them through proper design to ensure a reliable, safe and lasting structural performance. An important step in the vibration design of an engineering structure is the evaluation of its vibration modal characteristics, such as natural frequencies and mode shapes. This modal information plays a key role in the vibration design of the structure when subjected to dynamics excitations.

Beams are one of the most fundamental structural elements. A beam is typically described as a structural component having one dimension relatively greater than the other dimensions. Almost every engineering structure contains one or more beam components, such as bridges, helicopter blades and robot arms. In addition, many structures can be modeled at a preliminary level as beams. For example, a spring board or support of a wind power generation can be treated as a cantilever beam, and a span of an overhead viaduct or bridge can be viewed as a simply supported beam. A thorough understanding of the vibration characteristics of beams is of great significance for engineers to predict the vibrations of the whole structures.

In recent decades, many conventional beams used in the engineering applications are gradually being substituted by composite laminated materials due to their advanced material properties, including high strength-weight and stiffness-weight ratios, excellent vibration characteristics and good fatigue properties. Researches on the vibration and dynamic analyses of composite laminated beams have been increasing rapidly in the last two decades. A paper which reviewed most of the researches done in years 1989–2012 on the vibration analysis of composite beams by

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Hajianmaleki and Qatu [1] showed that research articles on the subject during period 2001–2012 are more than twice than those of 1989–2001. Due to the great importance, this paper considers the vibrations of composite laminated beams.

Composite laminated beams can be straight and curved. Generally, straight beams can be considered as a special case of curved beams with infinite radius of curvature (zero curvature). The vibrations of straight composite laminated beams with different boundary conditions have been studied intensively in the literature. Some of the more recent studies are those of Goyal and Kapania [2], Jafari-Talookolaei et al. [3], Li et al. [4–7], Khdeir [8], Chen et al. [9], Marur and Kant [10], Sakiyama et al. [11], Teboub and Hajela [12], Aydogdu [13], Vo and Thai [14,15], Khdeir and Reddy [16]. For the curved composite laminated beams (with different boundary conditions), however, only very limited references are available. Among those available, Li et al. [17] studied the vibration characteristics of laminated composite shallow circular arches by the dynamic stiffness method. Malekzadeh et al. [18,19] presented a differential quadrature solution for moderately thick laminated circular beams with general boundary conditions. Khdeir and Reddy [20] developed a model for the dynamic behaviors of laminated composite shallow arches from shallow shell theory. Tseng et al. [21] studied the free vibration of composite laminated beams of variable curvature based on the Timoshenko-type curved beam theory. Hajianmaleki and Qatu [22] employed a rigorous first order shear deformation theory along with modified ABD parameters to analysis the static and free vibration behaviors of generally laminated deep curved beams. Qatu and Elsharkawy [23] presented exact solutions for laminated composite beams with deep curvature and arbitrary boundaries by Ritz method. It has also been of great interest for researchers to develop an accurate and efficient formulation which can be used to determine the vibration behaviors of beams. During the last decade, Erasmo Carrera proposed the Carrera Unified Formulation (CUF), which was first applied to plates and shells and then extended to beams. The CUF permits one to develop a large number of beam theories with a variable number of displacement unknowns by means of a concise notation and by referring to a few fundamental nuclei. Higher-order beam theories can be easily implemented on the basis of the CUF, and the accuracy of a large variety of beam theories can be established in a hierarchical and/or axiomatic vs. asymptotic sense. A modern form of beam theories can therefore be constructed in a hierarchical manner [24–27].

After the review of the literature in this subject, it appears that most of the articles deal with a method or technique that is only suitable for a particular type of classical boundary conditions, i.e., simply-supported supports, clamped boundaries and free edges, which typically requires constant modifications of the solution procedures and corresponding computation codes to adapt to different boundary cases. This could become a very tedious work and result in repetitive programming and large computing cost due to the fact that the boundary conditions of a composite laminated beam may not always be classical in nature. A variety of possible boundary restraining cases, including classical boundary conditions, elastic restraints and their combinations may be encountered in the engineering practices. Therefore, it is necessary and of great significance to develop a unified, efficient and accurate formulation which is capable of universally dealing with composite laminated beams with arbitrary lamination schemes and general boundary conditions.

To the best of the authors' knowledge, vibration analysis of composite laminated deep curved beams with general boundary conditions is not available in the literature. Hence, in view of the apparent void, the present paper presents an endeavor to complement the vibration analysis of composite laminated deep curved beams. The title, A series solution for the vibrations of composite laminated deep curved beams with general boundaries, illustrates the main aim of this paper, namely: to develop a solution which is capable of dealing with vibrations of composite laminated deep curved beams with arbitrary lamination schemes and general boundaries, including classical boundaries, elastic supports and their combinations, thus to provide a unified and reasonable accurate alternative to other analytical and numerical techniques.

Under the current framework, the modified Fourier series method together with the Hamilton's principle and the artificial spring boundary technique are adopted to derive the theoretical formulation. The general boundary conditions of the beam are realized by applying the artificial spring boundary technique and the equations of motion and the related boundary equations are derived via the Hamilton's principle based on the first-order shear deformation theory. Each beam displacement, regardless of boundary conditions, is expanded as a modified Fourier series composed of a standard cosine Fourier series and certain supplementary terms introduced to remove the potential discontinuities at the ends and thus ensure and accelerate the convergence of series expansion. The characteristic equations are then derived directly in the matrix form by solving the equations of motion by combining the associated boundary equations and the modified Fourier series. The convergence and accuracy of the present formulation are tested and validated by several numerical cases against the results in the literature. The effects of shear deformation and inertia rotary, deepness term, boundary conditions, lamination schemes, material and geometrical parameters are investigated in detail as well. Finally, some new results are presented to provide useful information for the future researches.

2. Theoretical formulations

2.1. The model

As shown in Fig. 1, a laminated deep curved beam with uniform thickness h, width b is selected as the model. The beam is characterized by its middle surface, in which R represents the mean radius of the beam and θ_0 denotes its included angle. To describe the beam clearly, we introduce the following coordinate system: the θ-coordinate is taken along the length of the beam, and β- and z-coordinates are along the width and thickness directions, respectively. u, v and w separately indicate the middle surface displacement variations of the beam in the θ, β and z directions. The beam is assumed to be composed of arbitrary composite layers, which are perfectly bonded together. The distances from the top and the bottom surfaces of the kth layer to the middle surface are represented by Z_{k+1} and Z_k accordingly.

2.2. Kinematic relations and stress–strain relations

Within the framework of the first-order shear deformation theory, the displacement and rotation components of an arbitrary point of the laminated curved beam can be expressed as:

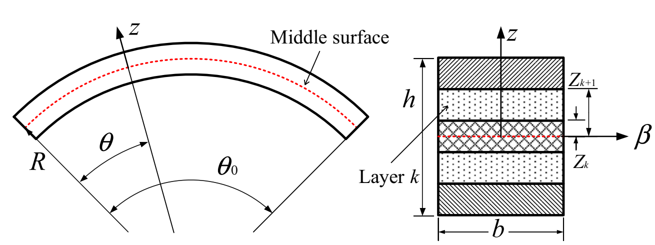

Fig. 1. Schematic diagram of composite laminated curved beams.