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Inverse estimation of heat flux and pressure in functionally graded cylinders with finite length

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ABSTRACT

An inverse algorithm is employed to estimate the time and spatially varying pressure and heat flux in functionally graded (FG) cylindrical shells with finite length by using the measured displacements and temperatures on their outer surfaces. The solution of corresponding direct problem is used to simulate the measured displacements and temperatures, which is obtained based on the three-dimensional thermoelasticity theory under axisymmetric conditions. As a powerful technique for optimization procedure of the inverse solution, the conjugate gradient method (CGM) together with the discrepancy principle is utilized. The governing differential equations subjected to the related boundary and initial conditions are discretized in both spatial and temporal domains by employing the differential quadrature method (DQM), as an efficient and accurate numerical tool. The good accuracy of the estimated internal pressure and heat flux is shown through different examples and by considering the influence of measurement errors, which validates the presented approach.

1. Introduction

Functionally graded materials (FGMs) are advanced heterogeneous materials with smooth and continuous spatial variation of the microstructures and compositions of their constituents. These materials can operate in high temperature thermal environments without losing their structural integrity [1]. FGMs are increasingly used in various fields of engineering such as aerospace, mechanical, energy, and nuclear engineering; for example, in rocket heat shields, heat exchanger tubes, extended surfaces (fins), thermoelectric generators, heat-engine components and plasma facings for fusion reactors [1,2].

Due to important practical applications of structural elements made of FGMs, accurate and efficient evaluation of their behaviors under thermo-mechanical loadings is essential for engineering design and manufacture. To do this, the applied thermo-mechanical loadings on the boundary of the body under consideration should be known. On the other hand, in many cases, the direct measurement of the thermo-mechanical boundary conditions requires the expensive measurement instruments or is quite complicated. As an alternative solution for this problem, one can use the inverse method in conjunction with simple measurement instruments to suitably estimate the unknown parameter(s) without perturbing the physical processes.

In recent years some interesting research works have been conducted on the inverse problems of the FGMs domains, which include heat transfer and thermoelastic analysis [3–14], and also material properties and shape identification [15–22]. Golbahar Haghighi et al. [11] presented the inverse two-dimensional heat transfer analysis of laminated cylindrical shells with FG layers to estimate the transient spatially varying heat flux on the inner surface of the shell. More recently, Golbahar Haghighi et al. [14] estimated the time-dependent internal pressure in a long FG cylindrical shell under thermal environment using the inverse method. The problem formulation is based on the one-dimensional elasticity theory and only the unknown internal pressure was obtained. In addition, there are some interesting research works on the static and dynamic analysis of FG shells; see for examples Refs. [23–30]. However, to the best of authors' knowledge, the simultaneous estimation of axially varying, transient internal pressure and heat flux in FG cylindrical shells using the inverse method is not investigated yet. Because of their practical importance and also challenging attraction for researchers in the field, this topic is considered in this research work.

It is well known that the inverse problems are mathematically ill-posed [31], and hence, an efficient optimization algorithm is required to solve these types of problems. On the other hand, it has been shown that the adjoint equation approach together with

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the conjugate gradient method (CGM) proposed by Alifanov [32] provide a very powerful technique for solving different inverse problems [5,6,8–14,33–37]. The main advantage of this method is that the regularization is performed during the iterative processes and therefore, the optimal regularization conditions are not required to be evaluated. Since in the CGM the searching direction depends on the gradient of the error functional of the problem under consideration, it may be evaluated by the related adjoint problem, and in addition, the optimal step size may be obtained from the related sensitivity problem [31,32,34].

Usually to validate an inverse algorithm, the solution of the related direct problem with some artificial errors is used as the measured data in the literature [8–14,33–37]. Hence, in verifying an inverse algorithm, accurate solution of the related direct problem is essential. But, since the three-dimensional thermoelastic governing equations of functionally graded cylindrical shells have variable coefficients, it is difficult to obtain an analytical solution. On the other hand, it has been shown that the differential quadrature method (DQM) as an accurate and efficient numerical tool can be employed for the analyzes of different problems of FG shells; see for example Refs. [38–42]. Therefore, in this work, this method is adopted to solve the governing equations of the inverse problem under consideration.

In the present work, the time and spatial dependent pressure and heat flux in FG cylindrical shells are estimated by employing the CGM in conjunction with the discrepancy principle. The measured displacements and temperatures on their outer surfaces are the only input data. The direct problem is formulated based on the three-dimensional thermoelasticity theory under axisymmetric loading conditions. The governing equations of the direct, adjoint and sensitivity problems under the related boundary and initial conditions are solved by using DQM. The versatility and robustness of the presented inverse algorithm are demonstrated through different examples. In the following, the fundamental parts of the present inverse algorithm are explained, and finally, the prepared numerical results are exhibited.

2. Direct problem

The geometry of FG cylindrical shell under consideration is shown in Fig. 1. In this figure, r and z represent the radial and axial cylindrical coordinate variables, respectively. The cylinder has an inner radius R_{in}, outer radius R_{out} and length L. Since the axisymmetric thermo-mechanical loadings is considered in this work, a two-dimensional cylindrical coordinate system (r,z) is sufficient to label the material points of the FG shell. It is assumed that the FG cylinder is simultaneously subjected to a pressure $p(z,t)$ and

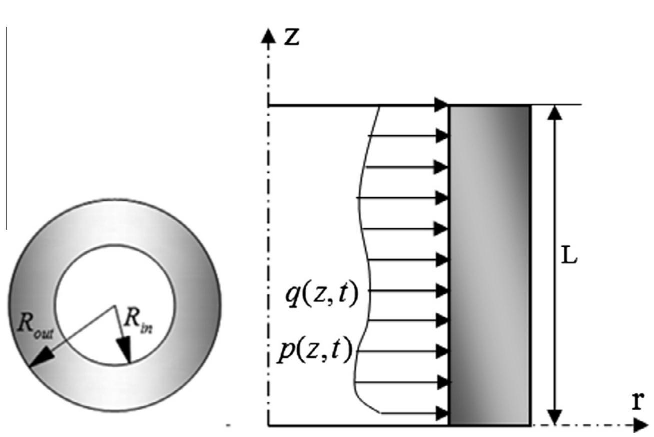

Fig. 1. The geometry, loading and coordinate system of the FG cylindrical shell under consideration.

heat flux $q(z,t)$ on its inner surface while the outer surface is traction free and also exchange heat with the surrounding media via the convection heat transfer phenomena. Hereafter, the parameter t stands the time variable.

Due to the axisymmetric thermal boundary and initial conditions, the temperature distribution in the FG cylinder can be find by solving the following two-dimensional governing differential equation subjected to the specified boundary and initial conditions,

$$\frac{\partial^2 T}{\partial r^2}+\left(\frac{1}{r}+\frac{1}{k}\frac{dk}{dr}\right)\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\hat{\alpha}}\frac{\partial T}{\partial t} \quad (1)$$

$$\text{At } r=R_{in}: -k\frac{\partial T}{\partial r}=q(z,t) \quad (2)$$

$$\text{At } r=R_{out}: k\frac{\partial T}{\partial r}+h(T-T_\infty)=0 \quad (3)$$

$$\text{At } z=0,L: \frac{\partial T}{\partial z}=0 \quad (4a,b)$$

$$\text{At } t=0: T(r,z,t)=T_0 \quad (5)$$

where $T[=T(r,z,t)]$ is the temperature, $k[=k(r)]$ the thermal conductivity, $\hat{\alpha}[=\hat{\alpha}(r)]=\frac{k}{\rho C}$ is the Fourier number, $\rho[=\rho(r)]$ the mass density, $C[=C(r)]$ the specific heat capacity, and $q(z,t)$ is the time and spatially varying heat flux on the inner surface of the cylinder, respectively. Also, T_0 is the initial temperature of the FG cylinder, which is assumed to be equal to the ambient temperature T_∞; h is the convection heat transfer coefficient on the outer surface of the cylinder.

The 3D thermoelastic equations of motion and the associated boundary conditions for an FG cylinder under axisymmetric loading and boundary conditions can be reduced to [40],

$$C_{11}\frac{\partial^2 u}{\partial r^2}+\left(\frac{dC_{11}}{dr}+\frac{C_{11}}{r}\right)\frac{\partial u}{\partial r}+C_{55}\frac{\partial^2 u}{\partial z^2}+\left(\frac{dC_{12}}{dr}-\frac{C_{11}}{r}\right)\frac{u}{r}+\frac{dC_{12}}{dr}\frac{\partial w}{\partial z}+(C_{12}+C_{55})\frac{\partial^2 w}{\partial r\partial z}-\rho\frac{\partial^2 u}{\partial t^2}=\left(\frac{dC_{11}}{dr}+2\frac{dC_{12}}{dr}\right)\varepsilon_T+(C_{11}+2C_{12})\frac{d\varepsilon_T}{dr} \quad (6)$$

$$C_{55}\frac{\partial^2 w}{\partial r^2}+\left(\frac{dC_{55}}{dr}+\frac{C_{55}}{r}\right)\frac{\partial w}{\partial r}+C_{11}\frac{\partial^2 w}{\partial z^2}+\left(\frac{dC_{55}}{dr}+\frac{C_{12}+C_{55}}{r}\right)\frac{\partial u}{\partial z}+(C_{12}+C_{55})\frac{\partial^2 u}{\partial r\partial z}-\rho\frac{\partial^2 w}{\partial t^2}=0 \quad (7)$$

At $z=0,L$:

$$\text{Either} \quad u=0 \quad \text{or} \quad C_{55}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)=0 \quad (8a,b)$$

$$\text{Either} \quad w=0 \quad \text{or} \quad C_{12}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)+C_{11}\frac{\partial w}{\partial z}=(2C_{12}+C_{11})\varepsilon_T \quad (9a,b)$$

At $r=R_{in}$:

$$C_{11}\frac{\partial u}{\partial r}+C_{12}\left(\frac{\partial w}{\partial z}+\frac{u}{r}\right)=(C_{11}+2C_{12})\varepsilon_T+p(z,t), \quad C_{55}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)=0 \quad (10a,b)$$

At $r=R_{out}$:

$$C_{11}\frac{\partial u}{\partial r}+C_{12}\left(\frac{\partial w}{\partial z}+\frac{u}{r}\right)=(C_{11}+2C_{12})\varepsilon_T, \quad C_{55}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)=0 \quad (11a,b)$$