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Structural design and experimental investigation on filament wound toroidal pressure vessels

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ABSTRACT

Composite toroidal pressure vessels have widely been used in many areas. In this paper a novel design approach for determining the winding parameters of filament wound toroidal pressure vessels was presented based on differential geometry and finite element method (FEM). The winding angle developments and the thickness distribution along the meridional direction were derived using the geodesic equation and fiber stability criteria. Then, a FEM-based design procedure was developed taking into account the variations of the laminate thickness and winding angle, in order to determine the optimal initial winding angle and lay-up. In addition, the rationality and accuracy of the present method was evaluated through the hydraulic test. The results show that the obtained fiber trajectories have good wind ability without fiber stacking or bridging. The results predicted using finite element simulation are consistent with the experimental data. Therefore, the present model and method provide an important reference for design and production of filament wound toroidal pressure vessels.

1. Introduction

Light-weight composite pressure vessels have gained widespread usage in fields of aerospace, automotive, underwater, energy storage and medical appliance [1,2]. Compared to traditional steel-based counterparts, composite pressure vessels show significant advantages, such as high specific stiffness/strength, outstanding fatigue properties, excellent corrosion resistance and good designability. As an anisotropic material, fibers can be deliberately oriented to the direction of the maximum principal stress and the vessel shape can be tailored to achieve the desirable vessel performance [3].

The determination of fiber trajectories reflects on one of the most key issues for the development of filament wound pressure vessels. Design and optimization of axisymmetrically convex shapes, such as cylinders, spheres and cones, have been extensively investigated by many researchers. Wang et al. [4–6] conducted research on stability condition of polar winding on the dished head, and predicted dome thickness distribution in the vicinity of the polar opening using a cubic spline function and finite element analysis. Zu et al. [7–9] developed several design and optimization methods for pressure vessels with unequal polar openings, articulated pressure vessels comprising various dome cells, and domed pressure vessels. Kim et al. [10] described an optimal design algorithm for filament wound cylinders based on the semi-geodesic path, progressive failure analysis and genetic algorithms. Kabir et al. [11] presented a numerical solution of cylindrical vessels with metallic liner, where the variable thickness around the dome area was considered. For most cylindrical shapes, the improvement of structural performance is severely limited due to the existence of end caps which leads to excessive fiber stacking and additional weight.

Compared to traditionally-used cylinders or spheres, the toroidal shape offers a promising alternative for producing light-weight pressure vessels. The torus is a doubly curved shape which can be regarded as a bent, endless cylinder that saves the materials on end caps [12]. The toroids also allow for various system components that can be stored through its central opening, which is essential for the space-limited storage [13]. However, the torus has a concave surface where the fibers may not be closely attached to the mandrel surface. A good windability is thus imperative for the design of filament wound toroids.

Toroidal pressure vessels have recently gained sufficient attentions, especially on the design and optimization of fiber trajectories and laminate parameters [9–14]. Chen [13] and Koussios et al. [14] designed fiber paths for toroidal vessels with the aid of the netting theory assuming that loads are only carried by fibers and fibers should thus be aligned to the principle stress directions. Koussios et al. [15] proposed a non-geodesic fiber trajectory for the design

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of toroids with a constant slippage coefficient. Zu et al. [16–18] proposed various design approaches for obtaining optimal geometries, fiber trajectories and winding patterns of filament-wound toroidal pressure vessels and developed a CAD system for their design and manufacturing. However, practical and effective design methods for toroidal pressure vessels have so far not been found. The netting theory is a simple method in which the contribution of resin matrix is negligible and its winding paths may lead to fiber bridging and slippage. In addition, it is difficult to accurately place the non-geodesics during the winding process, because the friction coefficient between the fibers and the supporting surface is always small which significantly reduces the design space of fiber trajectories [19].

Geodesic is the shortest path connecting any two points on a curved surface, which ensures the fiber stability during the winding process. The geodesic method which was first presented by Marketos [20] is applied here to obtain fiber trajectories avoiding fiber slippage and bridging. Compared to analytical solutions based on the netting theory or the classical lamination theory (CLT), FEM-based numerical solutions are more practical and effective, by which more structural specifics can be considered, such as the thickness variation caused by fiber stacking from the outside in over the torus. However, FEM-based design and experimental investigations on filament wound toroidal pressure vessels have been overlooked in previous research.

In this paper a reliable and accurate FEM design procedure was developed to determine the optimal geodesic-based toroidal pressure vessels. The geodesic equations for obtaining fiber trajectories on torus were derived using differential geometry and fiber stability-ensuring conditions. Then, a design method using the present FEM model was outlined, by which the variations of both the winding angle and the laminate thickness were taken into account. The optimal initial winding angle a_0 and corresponding layer number n for a toroidal pressure vessel was designed with the aid of the Tsai-Wu failure criterion. Finally, a hydraulic test was carried out to verify the accuracy and feasibility of the present method.

2. Geodesics on a torus

A toroid can be regarded as a complete shell with a circular area surrounding the rotation axis (Z-axis) through 360° (see Fig. 1). The toroid has a bending radius R and a tube's cross-sectional radius r. The vector representation of the toroid can be given by:

$$\vec{r}(\theta,\varphi) = \begin{Bmatrix} (R + r\cos\varphi)\cos\theta \\ (R + r\cos\varphi)\sin\theta \\ r\sin\varphi \end{Bmatrix}^T \tag{1}$$

where φ and θ stand for the angular coordinates along the meridional and parallel directions, respectively (see Fig. 1).The geodesic curvature of the torus can be given by *Liouville* formula:

$$k_g = \frac{d\alpha}{ds} + \frac{\sin\varphi}{R + r\cos\varphi}\cos\alpha \tag{2}$$

where a is the winding angle, s is the fiber length.

The geodesics represent the shortest paths connecting two arbitrary points on a continuous surface, and thus avoid any fiber slippage. Equalizing Eq. (2) to zero, the geodesic equation can be expressed as:

$$\frac{d\alpha}{ds} = -\frac{\sin\varphi}{R + r\cos\varphi}\cos\alpha \tag{3}$$

According to differential geometry, the following relations hold true:

$$\frac{d\theta}{ds} = \frac{\cos\alpha}{R + r\cos\varphi} \tag{4}$$

$$\frac{d\varphi}{ds} = \frac{\sin\alpha}{r} \tag{5}$$

Substituting Eq. (5) into Eq. (3) leads to:

$$\frac{d\alpha}{d\varphi} = -\frac{r\sin\varphi}{(R + r\cos\varphi)tg\alpha} \tag{6}$$

Substituting Eq. (5) into Eq. (4) leads to:

$$\frac{d\theta}{d\varphi} = \frac{r}{(R + r\cos\varphi)tg\alpha} \tag{7}$$

Integration of Eq. (6) with respect to φ, gives:

$$(R + r\cos\varphi)\cos\alpha = C \tag{8}$$

where C is the constant determined by the initial winding position & angle. With the aid of the initial winding conditions, the fiber trajectories can be obtained using *Runge–Kutta* method from Eqs. (6)–(8).

3. Stability-ensuring criteria during winding processing

Fiber bridging is one of the most important issues during filament winding process. Torus has a convex-concave surface and thus the winding tension induced by the feed eye may drive fiber out forward on the mandrel surface. In order to prevent fiber bridging, the normal curvature of the fiber path given by the *Euler* formula should be negative:

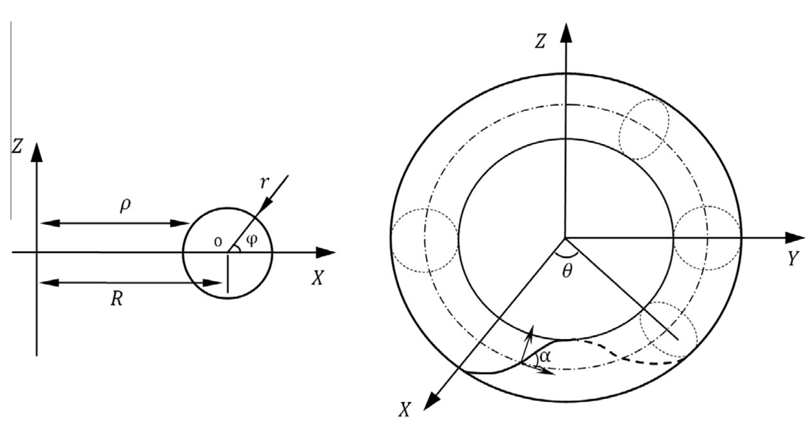

Fig. 1. Geometry of a toroid and its fiber trajectory.