Composite Structures 121 (2015) 225–236

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Stacking sequence optimization of composite laminates for maximum buckling load using permutation search algorithm

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ARTICLE INFO

Article history:
Available online 10 November 2014

Keywords:
Permutation search
Optimization
Stacking sequence design
Composite laminate
Repair strategy

ABSTRACT

Due to the involvement of large number of design variables, it is still one of the key concerns to build an efficient optimization algorithm for the stacking sequence design of composite laminates with various constraints. In this work, the flexural stiffness parameters are expressed in terms of the ply orientations, which helps to formulate the maximum of buckling load factor as a problem of identifying the optimum ply orientation at stacking positions. Afterwards, we suggest a permutation search (PS) algorithm to reduce the evaluations in stacking sequence optimization of composite laminates. In the first stage, permutation operations are sequentially performed for each permutation position, and in the second stage a repair strategy is adopted for overcoming the violation of constraints while maximizing the value of the objective function. A comparison has been performed between the PS and three genetic algorithm (GAs) methods. It has been demonstrated that the number of process analyses for stacking sequence optimization are greatly reduced by the PS algorithm. The novel PS algorithm combined with the modified repair strategy outperforms the studied GA methods for constrained stacking sequence optimization of composite laminate both in computational performance and finding the optimal objective value with high reliability.

1. Introduction

Due to the high specific strength and stiffness, composite materials have become widespread during the last three decades. The variation of thickness, fiber orientation as well as stacking sequence provides a large possibility of tailoring for achieving the required mechanical properties, such as in-plane, flexural and buckling behavior of composite laminates, which also makes the optimization of composite laminates has attracted increasing attention. The common problem is to develop optimization routines to minimize the weight of composite structures subjected to mechanical, blending and manufacturing constraints. In optimization studies, the flexural stiffness may be formulated as a linear function of lamination parameters and material invariants. Moreover, the lamination parameters are often treated as independent design variables and they can be expressed as the trigonometric functions of ply orientation, which are interrelated through the functions of stacking sequence [1–4]. Thus, the goal is often achieved by formulating the design of composite laminates as an optimization problem with stacking sequence and ply orientation considered as design variables. However, extremely high dimensional mixed-integer variables make it a challenging work to build an efficient optimization model to find the global optimum [2].

The problem of optimal design of composite laminates has been investigated by many researches. An extensive review of the topic can be found in a recent paper by Ghiasi et al. [3]. Due to its simple coding, escaping of gradient calculations, suitability for large variety of problems and capable of more likely to find global optima, genetic algorithm (GA) become the most popular heuristic method for stacking sequence optimization of composite laminates [2,3]. While it does not require the gradient or sensitivity coefficient evaluations, the population-based evolutionary algorithm can be computationally time consuming and expensive since large number of generations are usually required before converging to the optimal solution and each generation may consists of a large number of evaluations. Another major concern associated with GA is the premature convergence, which may happen if the initial population is not appropriately selected [3].

To reduce computing cost by decreasing the evaluation time and to increase the convergence rate by reducing the risk of premature convergence, many modifications have been suggested, such

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http://dx.doi.org/10.1016/j.compstruct.2014.10.031
0263-8223/

as the multi-level methods [5], the parallel computing method [5–7], and the Hybrid GA method [8,9], etc. In addition, some problem-dependent operators have been introduced to modify the standard genetic operators [10–15]. In stacking sequence optimization problems, some permutation operators are considered, such as the "gene swap", the "inversion" and the "mutation", etc. [16–20]. It is proved that these permutation operations can handle more effectively the search for an optimal stacking sequence by reducing the number of generations to convergence and the dimensionality of the design space [15,16].

However, in some optimization problems, the decision of permutation or sort itself is the main objective [21–23], such as the permutation optimum design of composite wing structure [4,16,24]. In wing structural optimization, the wing structure is often customarily divided into panels or regions, on which constraints from the overall structural design are imposed. Because of the industrial requirements and practical manufacturing considerations, the layer thickness for each panel or region is usually fixed and the fiber orientation angles are often limited to a discrete set. Thus, the optimization of the overall wing structure often specifies or imposes various constraints, on individual panel, such as the number of 0°, ±45°, 90° plies and in-plane loads. As a result, the optimization design is limited to stacking sequence permutations of given plies with fixed ply numbers of each orientation. Since GA is originally developed for unconstrained optimization problem, whereas the optimal design of stacking sequence is usually constrained by limitations in material's strength, weight, cost or other criteria, that must be incorporated. In order to add the constraints to the objective function of constrained permutation optimal problems, penalty function strategy is the most popular technique which may not be satisfied strictly [8,12], and may result in high cost of dealing with constraints. At the same time, the use of standard crossover operator in the permutation optimization problem may lead to inadmissible design individuals, which may then result in the variation of the ply numbers of each orientation. Therefore, appropriate operation, such as "gene repair" or "fixing-up" operation, is required to handle the constraint of ply numbers. Meanwhile, additional operations are also necessary for permutation if some other design constraints are assigned, such as the contiguity requirement [15,16]. The need for additional operations and the high cost of dealing with constraints via penalty function lead to the development of repair strategy to handle the constraints [16,18]. However, since the efficiency of GA is often sensitive to operations or parameters, unsuitable operations or parameters during the process of permutation and repair may result in heavy computational cost or even failure in finding the optimal solution [15].

In this study, based on the search or sort strategies of sequential search [21] and the selection sort [23] methods, a permutation search (PS) algorithm as well as an improved repair strategy is proposed to improve the numerical efficiency of the constrained sequence optimization of composite structures. The proposed algorithm takes the permutation of the ply angles as design variable, which governs the buckling behavior of composite laminates. Meanwhile, the number-of-ply constraints are handled by the improved repair strategies. The accuracy and computational efficiency of the present PS algorithm are then compared with three GA methods.

2. Optimization formulation

Considering the industrial and manufacturing requirements, symmetric and balanced stacking sequences are usually adopted in design, where the ply angles are pre-assigned and loading conditions of each panel are often specified. In this study, a symmetric laminated plate of rectangular shape composed of 0°, 45°, −45°, and 90° is examined under in-plane loading condition. The plate is simply supported at the edges and subjects to normal loads per unit length fF_x and fF_y in X and Y directions, respectively, and a shear load per unit length fF_{xy}, where f is a scalar amplitude parameter to the reference load F. (Fig. 1a). We consider the N plies laminate plate with its length (X direction) and width (Y direction) dimensions represented by a and b. For symmetric laminates, only $N/2$ plies are necessary to characterize the entire laminate, as shown in Fig. 1b. Again, due to the symmetry and balanced laminate, the extensional–flexural and the shear–extensional couplings can be eliminated. Here, the panel is designed to maximize the buckling load subject to a constraint on the number of plies of each orientation.

To simplify the optimization of the problem, we first take the normal and the shear loading conditions in consideration separately and then focus on the combined effects of different loading conditions. For a palte subject normal load, the governing differential equation for the bending of the panel reads [25]

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = F_x\frac{\partial^2 w}{\partial x^2} + F_y\frac{\partial^2 w}{\partial y^2} \quad (1)$$

Solution to Eq. (1) can be obtained via direct approach, such as the Navier approach and has the following general form for various boundary conditions

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\overline{X}_m(x)\overline{Y}_n(x) \quad (2)$$

where the functions $\overline{X}_n(x)$ and $\overline{Y}_n(x)$ are a set of functions that satisfy the boundary conditions, which are uniformly convergent, complete and orthogonal. Practically, because of uniform convergence, a finite number of terms are sufficient to provide any desired accuracy. Note that solutions for different boundary conditions can be obtained by defining corresponding forms of functions $\overline{X}_n(x)$ and $\overline{Y}_n(x)$ and substituting into Eq. (2).

For a plate that is simply supported on all four edges, we have

$$\text{at } x=0 \text{ and } x=a \quad w = M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} = 0 \quad (3)$$

$$\text{at } y=0 \text{ and } x=b \quad w = M_y = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} = 0 \quad (4)$$

Then Eq. (2) can be written as [26]

$$w(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \quad (5)$$

where m and n are integers. Under in-plane biaxial loading, the plate may buckle into m and n half waves along the length and width directions, respectively.

Substituting Eq. (5) into the governing partial differential Eq. (1), the load multiplier f can be expressed as [19]:

$$\lambda_{c,n}(m,n) = \pi^2\frac{D_{11}\left(\frac{m}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4}{F_x\left(\frac{m}{a}\right)^2 + F_y\left(\frac{n}{b}\right)^2} \quad (6)$$

where, the coefficients of the flexural stiffness D_{ij} (i, j = 1, 2, 6) depend on the lamination sequence.

The critical buckling load factor $\lambda_{c,n}$ is given by the minimum of $\lambda_{c,n}$ (m, n) over possible values of m and n, which varies with the total number of plies, the plate geometry, and the loading case. Herein, considering the insignificant contributions of coupling coefficients D_{16} and D_{26} special attention will be paid to D_{11}, D_{12}, D_{22} and D_{66}. For a composite laminate panel made of a single fibrous material, the elements of the flexural stiffness matrix can