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An orthotropic material model for steel fibre reinforced concrete based on the orientation distribution of fibres

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ABSTRACT

In the present paper the authors focus on constitutive mappings for steel fibre reinforced concrete, SFRC. The anisotropic properties of this composite are caused by the orientation distribution of fibres. The constitutive relation is developed for one meso-volume element of SFRC as a combination of isotropic and orthotropic St. Venant–Kirchhoff material models, which are applied to concrete matrix and to steel fibres, respectively. The alignment tensors and orientation distribution function adopted from the mesoscopic continuum theory are utilised to identify the material meso-symmetry axes and to asses the contribution of fibres in the symmetry axes defined. While assessing the orthotropic meso-elasticity for fibres, the elasticity of an individual fibre in its local coordinates is transformed into the material meso-symmetry axes and weighted with the orientation distribution function of fibres. The advantage of the material model developed for SFRC is that it uses complete orientation information of fibres (two angles in spherical coordinates) and utilises tensor quantities complying with material objectivity.

1. Introduction

Examples of natural materials with anisotropic material properties are wood, liquid crystals, soft tissue, examples of artificial ones include fibre-reinforced composites with a great variety of matrix and fibre materials. The development of artificial composites is often motivated by the need to increase material strength in an efficient and economically reasonable way. The aim of adding fibres to the matrix is the improvement of specific mechanical properties leading, however, often to anisotropic behaviour, which can be observed in reinforced rubber-like materials [28], in fibre-reinforced polymers [11,27,35], in carbon-reinforced composites [17], and paper [10]. The present research focuses on a cementitious composite formed by the mixing of concrete matrix with steel fibres, SFRC. The motivation to study this composite comes from the demand for using it as a material of load- bearing structures to reduce the construction time and improve the quality of structures. Despite the modelling of the properties of this composite is still a subject for discussions and research in engineering community [42], it is already quite extensively employed in the construction industry, for example, in floors resting on soil [24] and even in some load-bearing structures, such as elevated floor-slabs [6,41]. The complexity of SFRC also involves the presence of anisotropic behaviour occurring due to the different alignments of fibres. For example, when the alignment of fibres coincides with a principal stress in a structure, the contribution of fibres to material strength is more pronounced than otherwise. Although, the use of the orientation distribution function for short fibres dates back as far as 1952 [10], when a two-dimensional case was extensively analysed, the material models available for concrete reinforced by short fibres usually either consider the orientation of fibres utilising only a one-dimensional case with aligned fibres [5,40] or assume a mean orientation with respect to a predefined axis and use one orientation angle as a parameter [18,25]. One approach is the orientation number (ON), which is defined as an average projected length of fibres in a cross-section onto the normal of the cross-section divided by the fibre length [18]. Another approach is the orientation profile (OP) [25], which extends the concept of the orientation number counting the amount of fibres (out of the total number of fibres given) within different inclination intervals assuming a pre-defined statistical distribution. An alternative would be the use of full orientation information of fibres and tensor quantities, as it is done for other

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materials containing or consisting of orientable particles [1,2,12,14,20,45,46]. In spherical coordinates the position of a point is specified by three numbers: radial distance, inclination angle, azimuth (in-plane) angle. For the description of the orientation of a fibre two angles are necessary and the radius is not needed.

Within the present research, an orthotropic linear-elastic constitutive model for one meso-volume element of SFRC subjected to small (infinitesimal) deformations is developed. The model utilises the full orientation information of fibres (two angles in spherical coordinates), and employs tensor quantities. For describing the alignment of steel fibres the characteristics of the mesoscopic continuum theory are used: 2nd order alignment tensor to identify the material meso-symmetry axes and orientation distribution function to estimate the contribution of fibres in the symmetry axes defined [1,12,32].

The structure of the material model developed is based on a hyperelastic material. First, the strain-energy density function was considered as a function of the *Lagrangian* strain tensor [33]. To keep the material symmetry of orthotropic or isotropic hyperelastic material, the strain-energy density function should be given as an isotropic tensor function [21]. In turn, the isotropic tensor function can be represented as a function of its principal traces. The reasoning continues by the introduction of structural tensors enabling to lay down the material symmetry, and results in the isotropic tensor function of arguments among which are the structural tensors. In case of isotropic material symmetry the structural tensors are vanishing for the lack of preferred directions. For the orthotropic material symmetry the structural tensors are composed using the eigenvectors of the 2nd order alignment tensor representing the dominating directions of fibres. Thereby, the eigenvectors specify the material symmetry axes of one meso-volume element of SFRC. Onward, the quadratic terms of isotropic tensor function are utilised, thus leading to the orthotropic and isotropic St. Venant–Kirchhoff models, which, being differentiated, result in the 2nd Piola–Kirchhoff stress tensors. The advantage of using the 2nd Piola–Kirchhoff stress tensor for the case of linearised-elasticity is its symmetry and the differentiation gives the *second* elasticity tensor (4th order elasticity tensor). While assessing the orthotropic meso-elasticity for fibres, the longitudinal elasticity of an individual fibre in its local coordinates is transformed into the structural reference frame and weighted with the orientation distribution function of fibres. As a result, the orientation-weighted orthotropic meso-elasticity of a fibre in the structural frame is received. Further, this elasticity is transformed into material meso-symmetry coordinates specified by the eigenvectors of the 2nd order alignment tensor, which makes it possible to formulate the constitutive relation for one meso-volume element of SFRC in the material meso-symmetry system of coordinates using the elasticity constants weighted with the orientation distribution function of fibres.

In the application presented, the calculated fibre orientation parameters—2nd order alignment tensor and orientation distribution function of fibres—are based on experimentally measured fibre orientation distributions. The experimental samples were extracted from full-size floor-slabs (Section 6) and analysed by X-ray micro-computed tomography (μCT). The orientation of each individual fibre was measured and the orientation state of all fibres in a sample were computed. The detailed description of the application of μCT method for measuring the parts of SFRC structures and the analysis of outcomes are presented in [39].

2. Basic notations and definitions

Vectors and tensors are either denoted by bold letters or using index-notation for components with respect to an arbitrary fixed basis, for shortness the basis vectors will be omitted from the equations. Explicit calculations are performed in Cartesian coordinates. In index-notations, the Einstein summation convention is used. A summary of the used symbols and notation is given in Table 1.

3. Behaviour of SFRC

The tensile strength of SFRC depends on the alignment of fibres in the concrete matrix [4]. In a tensioned SFRC member, where all fibres are aligned with each other, as well as with the principal tensile stress, the fibres have an optimal orientation and thus contribute to structural tensile capacity with the highest efficiency. In a typical concrete member, which is reinforced by steel bars in the direction of expected tensile stresses, the stress behaviour has an orthotropic character. A similar situation may be in a SFRC member and the problem is to identify and model the directions of fibre alignments. The measurements of fibre orientations from the samples extracted from full-size floor slabs, utilised in the present research, revealed the variations of fibre orientation distributions along the all (X, Y, Z) axes of the slabs. The latter indicates that a theory capable to consider the spatial—three-dimensional—nature of SFRC material properties is needed. Thus, the constitutive relation for SFRC is justified to be developed based on orthotropic material model. The detailed outcomes of the measurements of fibre orientations and the features of fibre alignments are presented in [13,39].

In addition, a linear dependence between stresses and deformations is assumed. Let us examine a bended concrete member. In general, compressed concrete is an elasto-plastic material, where simultaneously both elastic and plastic deformations are developing. As a consequence, the relation between the stress and deformation should be non-linear. In a bended concrete, until the first cracks have appeared in the tension zone of a cross-section the relation between the stress and deformation can be considered as linear [3], Fig. 1.

As soon as cracks are occurring in the tension zone, the deformations start to grow rapidly and the member breaks suddenly in a brittle manner. Accordingly, SFRC has a similar brittle character since the failure regime of SFRC is largely determined by the mode of the loss of bond strength on fibre-concrete interface, and this is rather brittle than ductile [47]. The brittle behaviour

Table 1
Summary of used symbols and notation [15,33].

$\mathbf{v}, v_i$	vector (bold small letters)
$\mathbf{A}, A_{ij}$	2nd order tensor
${}^{\langle l \rangle}\mathbf{A}, A_{\mu_1 \ldots \mu_l}$	l-order tensor
$\mathbf{I}, {}^{\langle 4 \rangle}\mathbf{I}$	2nd and 4th order identity tensors
$\mathbf{v} \otimes \mathbf{n} = \mathbf{A},\ v_i n_j = A_{ij}$	outer product of two vectors
$\underbrace{\overline{\mathbf{n} \otimes \ldots \otimes \mathbf{n}}}_{\text{l−times}}$	l-order symmetric irreducible (traceless) part of an l-order symmetric tensor formed by the l-order outer products of a vector $\mathbf{n}$ with itself
$\mathbf{AB} = \mathbf{A} \cdot \mathbf{B} = A_{ik} B_{kj}$	inner product (also called scalar- or dot-product)
$\mathbf{A} \tilde{\otimes} \mathbf{B} \hat{=} A_{ij} \tilde{\otimes} B_{kl} = M_{ilkj}$	modified outer product
$W = W(\mathbf{F})$	strain-energy density function
$\mathbf{F}$	deformation gradient
$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I})$	*Lagrangian* strain tensor
$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\mathbf{F} + \mathbf{F}^T\right) - \mathbf{I}$	infinitesimal strain tensor
$\mathbf{S} = \frac{\partial W}{\partial \mathbf{E}}$	2nd *Piola–Kirchhoff pseudo-stress* tensor
${}^{(c)}, {}^{(m)}, {}^{(f)}, {}^{(s)}$	refer to composite, matrix, fibres, steel, respectively
${}^{ij}, {}^{i}, {}^{j}, \ i,j = 1,2,3$	upper indices refer to material symmetry axes
${}^{(f_{ms})}, {}^{(f_{str})}$	refer to orientation-weighted fibres in material meso-symmetry and structural coordinates, respectively.
^{S}	refers to symmetric part of a tensor (minor symmetry within last 2 indices)