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Free vibration of simply supported and multilayered magneto-electro-elastic plates

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ABSTRACT

Based on three-dimensional elasticity theory, semi-analytical solutions for free vibration of simply supported and multilayered magneto-electro-elastic plates have been derived applying a newly developed hybrid analysis, which combines the state space approach (SSA) and the discrete singular convolution (DSC) algorithm. The thickness direction of plate is selected as the transfer direction in SSA, and the DSC is applied to discretize the in-plane domains. Hence, the original partial differential equations are translated into a state equation consisting of first-order ordinary differential equations. The application of DSC makes it possible to treat various boundary conditions, and shows excellent performance for high frequency vibration. The accuracy and convergence of SS-DSC is validate through numerical examples.

1. Introduction

With the increasing technological applications of piezoelectric and piezomagnetic materials in smart structures, the problems associated with magneto-electro-elastic materials have gained considerable attention recently [1–5]. There exhibit a coupling effect between electric and magnetic fields in these materials, which can be applied in smart structures. These materials have the ability to convert energy from one form to another, for example magnetic, electric and mechanical energy. Researches on static and dynamic behavior of magneto-electro-elastic materials have been done in literature.

Pan [6] derived an exact three-dimensional solution of simple supported multilayered magneto-electro-elastic plate. Wang [7] applied the state space formulations [8,9] to study the bending of multi-layered magneto-electro-elastic rectangular plates. Chen and Lee [10] presented novel state space formulations for the static problem of transversely isotropic thermo-magneto-electro-elastic materials using a separation technique. Pan and Heyliger [11] found that some natural frequencies of a multi-field plate were identical to the ones of the corresponding elastic plate. Nonlinear free vibration of magneto-electro-elastic rectangular plates is studied by Razavi [12]. Wang [13] presented state vector approach of free-vibration analysis of magneto-electric-elastic hybrid laminated plates.

Some studies have been done on magneto-electro-elastic structures by finite element method. Buchanan [14] has studied the free vibration of infinitely long magneto-electro-elastic cylindrical shell using semi-analytical finite element method. Buchanan [15] also studied the layered versus multiphase magneto-electro-elastic finite long plate composites. A layerwise partial mixed finite element model has been developed by Lage [16] for static analysis of magneto-electro-elastic plate. Bhangale and Ganesan [17,18] proposed the finite analysis for FGM cylindrical shells. Bhangale and Ganesan [19] also presented static analysis of FGM magneto-electro-elastic plate by finite element method.

Discrete singular convolution (DSC) algorithm was introduced by Wei [20] in 1999. Singular convolutions are a special class of mathematical transformations which appear in many science and engineering problems, such as Hilbert, Abel and Radon transforms. These transforms are essential to many practical applications, such as computational electro-magnetics, signal and image processing, pattern recognition, tomography, molecular potential surface generation and dynamic simulation. It is the most convenient way to discuss the singular convolution in the context of the theory of distributions. It not only provides a rigorous justification for a number of informal manipulations in physical science and engineering, but also opens a new area of mathematics, which in turn gives impetus in many other mathematical disciplines, such as operator calculus, differential equations, functional analysis, harmonic analysis and transformation theory [21]. The theory of wavelets and frames, a new mathematical branch developed in recent years, can also find its root in the theory of distributions [22].

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Wang [23] studied simply supported anisotropic rectangular plate by DSC method. Nonlinear static response of laminated composite plates is analyzed by using DSC method [24]. Ö Civalek [25] studied vibration of isotropic conical shells applying DSC. Ö Civalek [26] presented vibration analysis of conical panels using DSC method. A four-node DSC for geometric transformation was applied to investigate vibration of arbitrary straight-sided quadrilateral plates [27].

Wei [28] and Zhao et al. [29] analyzed the high frequency vibrations of plates and plate vibration under irregular internal support using DSC algorithm. Wan et al. [30] studied the unsteady incompressible flows using DSC. Ng et al. [31] presented a comparative accuracy of DCS and generalized differential quadrature methods for vibration analysis of rectangular plates. Yunshan [32] applied the DSC-Ritz method to study the free vibration of Mindlin plates. Lim et al. [33] proposed the DSC-Ritz method for high-mode frequency analysis of thick shallow shells. It is showed that the DSC algorithm works very well for the vibration analysis of plates, especially for high-frequency analysis. It is also concluded that the DSC algorithm had global methods' accuracy and local methods' flexibility for solving differential equations.

In this paper, the hybrid semi-analytical elasticity method (SS-DSC) is introduced to the analysis of multilayered magneto-electro-elastic plates. The thickness direction of the laminates is treated as the transfer direction in the SSA, while DSC method is applied to discretize the in-plane domains, thereby showing accuracy for high frequency vibration problems. The transfer relation of state vector between layers is established by incorporating continuity conditions at interfaces. Numerical examples are performed to validate the present method, and present global methods' accuracy and local methods' flexibility.

2. Basic equations

For a transversely isotropic magneto-electro-elastic medium in Cartesian co-ordinate system, the coupled constitutive equations can be written as [6]

$$\begin{aligned} \sigma_j &= C_{jk}S_k - e_{kj}E_k - q_{kj}H_k, \\ D_j &= e_{jk}S_k + \varepsilon_{jk}E_k + m_{jk}H_k, \\ B_j &= q_{jk}S_k + m_{jk}E_k + \mu_{jk}H_k, \end{aligned} \tag{1}$$

where σ_j denotes stress, D_j is electric displacement and B_j is magnetic induction. C_{jk}, ε_{jk} and μ_{jk} are the elastic, dielectric and magnetic permeability coefficient. e_{jk}, q_{jk} and m_{jk} are piezoelectric, piezomagnetic and magnetoelectric material coefficients.

The strain displacement relations are

$$\begin{aligned} &S_{xx} = \frac{\partial u}{\partial x}, \quad S_{yy} = \frac{\partial u}{\partial y}, S_{zz} = \frac{\partial u}{\partial z}, \\ &S_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \quad S_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad S_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \end{aligned} \tag{2}$$

where u, v and w are mechanical displacement in co-ordinate directions x, y and z.

The electric field vector can be expressed by the electric potential as follows:

$$E_x = -\frac{\partial \phi}{\partial x}, \quad E_y = -\frac{\partial \phi}{\partial y}, \quad E_z = -\frac{\partial \phi}{\partial z}. \tag{3}$$

The magnetic field vector can be expressed by the magnetic potential as follows:

$$H_x = -\frac{\partial \psi}{\partial x}, \quad H_y = -\frac{\partial \psi}{\partial y}, \quad H_z = -\frac{\partial \psi}{\partial z}. \tag{4}$$

3. State vector formulation

The state vector approach is based on the mixed formulation of solid mechanics in which u, v, w, σ_z, D_z, B_z, τ_{zx}, τ_{zy}, ϕ and ψ are taken as basic unknowns. Following the process of state vector approach in piezoelasticity and eliminating from the governing equations (1)–(4), the field equations can be recast in the following matrix form [6]:

$$\frac{\partial \eta_1}{\partial z} = \mathbf{A}\eta_1, \quad \eta_2 = \mathbf{B}\eta_1, \tag{5}$$

where η_1 is the basic unknown vector, which is called the state vector. η_2 is related to η_1 by Eq. (5).

$$\eta_2 = [\sigma_x \quad \sigma_y \quad D_x \quad D_y \quad B_x \quad B_y]^T,$$

$$\eta_1 = [u \quad v \quad D_z \quad B_z \quad \sigma_z \quad \tau_{zx} \quad \tau_{zy} \quad \phi \quad \psi \quad w]^T, \tag{6}$$

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{A}_1 \\ \mathbf{A}_2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \mathbf{B}_1 & 0 \\ 0 & \mathbf{B}_2 \end{bmatrix}, \tag{7}$$

$$\mathbf{A}_1 = \begin{bmatrix} a & 0 & -\beta_1 p_1 & -\gamma_1 p_1 & -p_1 \\ 0 & b & -\beta_4 p_2 & -\gamma_3 p_2 & -p_2 \\ -\beta_1 p_1 & -\beta_4 p_2 & \beta_2 p_1^2 + \beta_5 p_2^2 & \beta_3 p_1^2 + \beta_6 p_2^2 & 0 \\ -\gamma_1 p_1 & -\gamma_3 p_2 & \beta_3 p_1^2 + \beta_6 p_2^2 & \gamma_2 p_1^2 + \gamma_4 p_2^2 & 0 \\ -p_1 & -p_2 & 0 & 0 & \rho p^2 \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} \rho p^2 - (\alpha_1 p_1^2 + C_{66} p_2^2) & -(\alpha_2 + C_{66}) p_1 p_2 & -\alpha_4 p_1 & -\alpha_5 p_1 & -\alpha_3 p_1 \\ -(\alpha_6 + C_{66}) p_1 p_2 & \rho p^2 - (\alpha_7 p_2^2 + C_{66} p_1^2) & -\alpha_9 p_2 & -\alpha_{10} p_2 & -\alpha_8 p_2 \\ -b_{21} p_1 & -b_{22} p_2 & a_4 & a_5 & a_2 \\ -b_{31} p_1 & -b_{32} p_2 & a_5 & a_6 & a_3 \\ -b_{11} p_1 & -b_{12} p_2 & a_2 & a_3 & a_1 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} \alpha_1 p_1 & \alpha_2 p_2 & \alpha_4 & \alpha_5 & \alpha_3 \\ \alpha_6 p_1 & \alpha_7 p_2 & \alpha_9 & \alpha_{10} & \alpha_8 \\ C_{66} p_2 & C_{66} p_1 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_2 = \begin{bmatrix} \beta_1 & 0 & -\beta_2 p_1 & -\beta_3 p_1 & 0 \\ 0 & \beta_4 & -\beta_5 p_2 & -\beta_6 p_2 & 0 \\ \gamma_1 & 0 & -\beta_3 p_1 & -\gamma_2 p_1 & 0 \\ 0 & \gamma_3 & -\beta_6 p_2 & -\gamma_4 p_2 & 0 \end{bmatrix}, \tag{8}$$

where

$$\begin{aligned} &\kappa a_1 = \varepsilon_{33}\mu_{33} - d_{33}^2, \quad \kappa a_2 = e_{33}\mu_{33} - d_{33}q_{33}, \quad \kappa a_3 = q_{33}\varepsilon_{33} - e_{33}d_{33}, \\ &\kappa a_4 = -C_{33}\mu_{33} - q_{33}^2, \quad \kappa a_5 = C_{33}d_{33} + e_{33}q_{33}, \quad \kappa a_6 = -C_{33}\varepsilon_{33} - e_{33}^2, \\ &\kappa = \varepsilon_{33}(C_{33}\mu_{33} + q_{33}^2) - d_{33}(C_{33}d_{33} + 2e_{33}q_{33}) + e_{33}^2\mu_{33}, \end{aligned}$$

$$\begin{aligned} &b_{11} = a_1 C_{13} + a_2 e_{31} + a_3 q_{31}, \quad b_{12} = a_1 C_{23} + a_2 e_{32} + a_3 q_{32}, \\ &b_{21} = a_2 C_{13} + a_4 e_{31} + a_5 q_{31}, \\ &b_{22} = a_2 C_{23} + a_4 e_{32} + a_5 q_{32}, \quad b_{31} = a_3 C_{13} + a_5 e_{31} + a_6 q_{31}, \\ &b_{32} = a_3 C_{23} + a_5 e_{32} + a_6 q_{32}, \end{aligned}$$

$$\begin{aligned} &\alpha_1 = C_{11} - C_{13}b_{11} - e_{31}b_{21} - q_{31}b_{31}, \\ &\alpha_2 = C_{12} - C_{13}b_{12} - e_{31}b_{22} - q_{31}b_{32}, \\ &\alpha_3 = C_{13}a_1 + e_{31}a_2 + q_{31}a_3, \quad \alpha_4 = C_{13}a_2 + e_{31}a_4 + q_{31}a_5, \\ &\alpha_5 = C_{13}a_3 + e_{31}a_5 + q_{31}a_6, \\ &\alpha_6 = C_{12} - C_{23}b_{11} - e_{32}b_{21} - q_{32}b_{31}, \\ &\alpha_7 = C_{22} - C_{23}b_{12} - e_{32}b_{22} - q_{32}b_{32} \\ &\alpha_8 = C_{23}a_1 + e_{32}a_2 + q_{32}a_3, \quad \alpha_9 = C_{23}a_2 + e_{32}a_4 + q_{32}a_5, \\ &\alpha_{10} = C_{23}a_3 + e_{32}a_5 + q_{32}a_6, \end{aligned}$$