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Analysis of cross-laminated timber by computational homogenisation and experimental validation

E.I. Saavedra Flores [a,*], I. Dayyani [b], R.M. Ajaj [c], R. Castro-Triguero [d], F.A. DiazDelaO [e], R. Das [f], P. González Soto [a]

[a] Departamento de Ingeniería en Obras Civiles, Universidad de Santiago de Chile, Av. Ecuador 3659, Estación Central, Santiago, Chile
[b] College of Engineering, Swansea University, Singleton Park, Swansea SA2 8PP, United Kingdom
[c] Aeronautics and Astronautics, University of Southampton, Southampton SO171BJ, United Kingdom
[d] Department of Mechanics, University of Cordoba, Campus de Rabanales, Cordoba CP 14071, Spain
[e] Institute for Risk and Uncertainty, School of Engineering, University of Liverpool, Brownlow Hill, Liverpool L69 3GH, United Kingdom
[f] Department of Mechanical Engineering, University of Auckland, Auckland 1010, New Zealand

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ABSTRACT

In this paper we investigate the mechanical behaviour of cross-laminated timber panels by a computational homogenisation approach. A finite element procedure is adopted within a multi-scale modelling framework to determine the constitutive response of timber. As some of the microstructural parameters of wood are either not well-known or susceptible to considerable variation, we introduce uncertainty in the definition of the material. In order to validate the present multi-scale model, we measure experimentally the longitudinal Young's modulus and density of sawn wood beams made of radiata pine. In addition, we carry out several experimental tests on cross-laminated timber panels subject to bending, shear and compression loads. Our numerical predictions are compared with the experiments and are validated successfully, revealing the potential predictive capabilities of the present multi-scale modelling for the analysis of wood materials and timber structures.

1. Introduction

In spite of the evident ecological advantages of building with wood, and the considerable growth that the total production is experiencing in the world market [1], the benefits of using wood in the building and construction sector are still far from being maximised. This is mainly due to the fact that dimensioning practices and many existing structural design rules are still based on an empirical background [2]. When compared to the structural design codes of more popular building materials, such as concrete or steel, whose codes and regulations have undergone a remarkable modernisation over the last few decades, standards for the design of timber structures are still in a very preliminary state of development [3]. This has inevitably led to discourage the use of timber in the construction sector.

The reason for the slow progress in the development of timber design codes, and in particular, in the difficulties to fully understand the mechanics of timber materials, lies mainly in the highly complex and intricate nature of wood microstructure. At very small scales, wood shows a complicated (but well organised) hierarchical nature distributed across multiple spatial scales, from submicrometer dimensions to macroscopic scales [4]. This important feature has been a subject of intensive research over the last few years by means of multi-scale homogenisation techniques. Holmberg et al. [5] studied the mechanical behaviour of wood by a multi-scale procedure incorporating growth rings, irregularity in the shape of cells and anisotropy in the layered structure of cell-walls. Hofstetter et al. [6,7] suggested five elementary phases for the mechanical characterisation of wood. These were hemicellulose, lignin, cellulose, with its crystalline and amorphous portions, and water. Qing and Mishnaevsky [8,9] proposed a model taking into account several scale levels and investigated the influence of microfibril angles, shape of the cell cross-section and wood density on the elastic properties of wood. Later, Qing and Mishnaevsky [10] extended their model by incorporating progressive damage to the homogenised elasticity matrix. Rafsanjani et al. [11] investigated the hygro-mechanical behaviour of growth rings by means of the computational homogenisation of wood at two scales. In Refs. [12,13], the authors investigated the non-linear irreversible behaviour of the wood cell-wall composite. A similar multi-scale approach was adopted to develop a new material [14], by following

* Corresponding author. Tel.: +56 (2) 27182803.
E-mail address: erick.saavedra@usach.cl (E.I. Saavedra Flores).

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a wood-inspired strategy [15,16]. Saavedra Flores et al. [17] studied the extensibility of the wood cell-wall composite by considering uncertainty in the material properties. Nevertheless, despite the increasing interest in this subject and the considerable effort devoted to the computational modelling of timber structures [18–21], the complete understanding of the mechanical properties of this material at small spatial scales, and their implications on the macroscopic response, is still an issue which remains open at present.

In recent years, cross-laminated timber (CLT) has become a new prefabricated constructive system, whose use has been increasingly spreading over Europe [22] and elsewhere, particularly in the construction sector. CLT panels are composite structures consisting of several layers of boards stacked crosswise and glued together on their faces. Different methods have been adopted for the determination of the basic mechanical properties of CLT, comprising analytical and experimental approaches. However, to date no method has been universally accepted by CLT manufacturers and designers [23]. Therefore, a profound knowledge about the mechanical behaviour of these products is necessary to improve and develop design concepts [24] and manufacture processes.

In order to establish a firm scientific basis for the better understanding of wood mechanics, and in particular CLT composite structures, the main objective of this paper is to investigate the mechanical behaviour of CLT panels by means of a multi-scale computational homogenisation approach. We validate the present multi-scale framework by comparing our numerical predictions with experimental data of CLT panels made of radiata pine subject to bending, shear and compressive loads.

2. Computational homogenisation

In the present type of homogenisation-based multi-scale theory it is assumed that the macroscopic or homogenised strain tensor $\boldsymbol{\varepsilon}$ at any arbitrary point $\boldsymbol{x}$ of the macroscopic continuum is the volume average of the microscopic strain tensor field $\boldsymbol{\varepsilon}_\mu$ over the domain Ω_μ of a representative volume element (RVE) of material. Similarly, the macroscopic or homogenised stress tensor field $\boldsymbol{\sigma}$ is assumed to be the volume average of the microscopic stress tensor $\boldsymbol{\sigma}_\mu$, over Ω_μ.

Furthermore, it is possible to decompose the local displacement field $\boldsymbol{u}_\mu$ over the RVE domain as a sum of a linear displacement $\boldsymbol{\varepsilon}(\boldsymbol{x})\boldsymbol{y}$, which represents a homogeneous strain, and a displacement fluctuations field $\tilde{\boldsymbol{u}}_\mu$. Here, $\boldsymbol{y}$ is the local RVE coordinate. The displacement fluctuations field represents local variations about the linear displacement $\boldsymbol{\varepsilon}(\boldsymbol{x})\boldsymbol{y}$ and does not contribute to the macroscopic scale strain.

By taking into account the Hill–Mandel Principle of Macro-homogeneity [25,26], which establishes that the macroscopic stress power must equal the volume average of the microscopic stress power over Ω_μ, the virtual work equation for the RVE can be reduced to

$$\int_{\Omega_\mu} \boldsymbol{\sigma}_\mu(\boldsymbol{y}) : \nabla^s \boldsymbol{\eta} dV = 0, \tag{1}$$

with $\boldsymbol{\eta}$ representing the virtual kinematically admissible displacements field of the RVE, and ∇^s the symmetric gradient operator.

In order to make problem (1) well-posed, a set of kinematical constraints upon the selected RVE is required. In what follows, the choice of this set will coincide with the widely used periodic boundary displacement fluctuations model [27], which is typically associated with the modelling of periodic media.

We must note that in Eq. (1) the virtual work of the RVE body force and surface traction fields vanish since they are the reaction forces associated to the set of imposed kinematical constraints. By taking this into account, and after a trivial tensorial manipulation, the macroscopic stress tensor $\boldsymbol{\sigma}$ can also be expressed as

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \frac{1}{V_\mu} \int_{\partial\Omega_\mu} \boldsymbol{t}(\boldsymbol{y}) \otimes_s \boldsymbol{y} dA, \tag{2}$$

with V_μ the volume of the RVE associated to the point $\boldsymbol{x}$, $\boldsymbol{t}$ the RVE boundary tractions, and $\otimes_s$ the symmetric tensorial product.

A generic implicit finite element discretisation scheme is used as the underlying framework. The first crucial component of the implicit finite element approximation consists of an incremental (time-discrete) counterpart of the original microscopic constitutive law. Within a time interval $\Delta t = t_{n+1} - t_n$, the microscopic stress tensor $\boldsymbol{\sigma}_\mu|_{n+1}$ at time t_{n+1} is determined as a function of the microscopic strain tensor $\boldsymbol{\varepsilon}_\mu|_{n+1}$ at time t_{n+1}. The following basic ingredient in the finite element approximation is the incremental form of the microscopic equilibrium Eq. (1). By introducing a time-discrete constitutive functional, $\hat{\boldsymbol{\sigma}}_\mu$, the incremental equilibrium problem of step $n+1$ can be obtained straightforwardly. In order to complete the numerical approximation of the model, a standard finite element discretisation h is introduced. By replacing the domain Ω_μ with its discrete counterpart Ω_μ^h, the fully spatial–temporal discretised version of Eq. (1) is obtained:

$$\int_{\Omega_\mu^h} \mathbf{B}^{\mathrm{T}} \hat{\boldsymbol{\sigma}}_\mu(\boldsymbol{\varepsilon}_{n+1} + \mathbf{B}\tilde{\mathbf{u}}_\mu|_{n+1}) dV \cdot \boldsymbol{\eta} = 0, \tag{3}$$

in which $\mathbf{B}$ denotes the global strain–displacement matrix, $\boldsymbol{\varepsilon}_{n+1}$ is the prescribed finite element array of macroscopic engineering strains at time t_{n+1}, $\hat{\boldsymbol{\sigma}}_\mu$ is the incremental constitutive functional at the RVE level that delivers the array of stress components, $\tilde{\mathbf{u}}_\mu|_{n+1}$ is the array of global nodal displacement fluctuations and $\boldsymbol{\eta}$ is the array of global nodal virtual displacements.

Within an infinitesimal linear elastic strain regime, the above algebraic Eq. (3) can be solved directly by rearranging it in standard fashion as

$$[\mathbf{F} + \mathbf{K}\tilde{\mathbf{u}}_\mu] \cdot \boldsymbol{\eta} = 0, \tag{4}$$

where $\mathbf{F}$ is a global nodal force vector, and $\mathbf{K}$ the stiffness matrix.

For convenience, we shall assume the finite element mesh topology here to be such that a one-to-one correspondence exists between nodes of opposing sides of the RVE boundary. We split the RVE mesh into three subsets of nodes: one set of interior nodes, with corresponding quantities denoted by the subscript i, and; two sets of boundary nodes denoted, respectively, with subscripts + and − so that for each node of set + with coordinate $\boldsymbol{y}_+$ there is an opposite node of set − with coordinate $\boldsymbol{y}_-$ satisfying the periodic constraint. In this case, the periodic kinematical constraint can be enforced by simply requiring each pair of such opposite nodes to have identical displacement fluctuation. That is, $\tilde{\mathbf{u}}_{\mu_+} = \tilde{\mathbf{u}}_{\mu_-}$.

By applying the same partition to the components of $\mathbf{F}$, $\mathbf{K}$, $\tilde{\mathbf{u}}_\mu$ and $\boldsymbol{\eta}$, and keeping in mind that $\boldsymbol{\eta}_+ = \boldsymbol{\eta}_-$ (for each pair of opposite points $\boldsymbol{y}_+$ and $\boldsymbol{y}_-$, respectively), the explicit solution of Eq. (4) for the nodal displacement fluctuations vector is given by

$$\begin{bmatrix} \tilde{\mathbf{u}}_{\mu_i} \\ \tilde{\mathbf{u}}_{\mu_+} \end{bmatrix} = -\begin{bmatrix} \mathbf{k}_{ii} & \mathbf{k}_{i+} + \mathbf{k}_{i-} \\ \mathbf{k}_{+i} + \mathbf{k}_{-i} & \mathbf{k}_{++} + \mathbf{k}_{+-} + \mathbf{k}_{-+} + \mathbf{k}_{--} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{F}_i \\ \mathbf{F}_+ + \mathbf{F}_- \end{bmatrix}. \tag{5}$$

With the global nodal displacement fluctuations vector $\tilde{\mathbf{u}}_\mu$ at hand, it is straightforward to compute the homogenised stress vector $\boldsymbol{\sigma}$ by using the discretised version of Eq. (2). Then, the calculation of the corresponding homogenised elasticity matrix $\mathbf{D}$ is trivial.

Finally, the computational homogenisation procedure described in this section is implemented in the commercial software ANSYS