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A variable metric stochastic theory of textile composites with random geometric parameters of yarn cross-section

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ABSTRACT

A variable metric stochastic theory of the elastic constants of textile composites with stochastic yarn (or tow) geometry has been developed in the current study. A variable metric coordinate transformation is utilized in this theory to introduce the cross-section shape fluctuations of yarn into the compliance of textile composites. Six parameters are defined to describe the positioning and cross-section shape of yarn completely. Then, the Taylor expansions of the local stochastic variable metric basis and the compliance of yarn are executed. Finally, the volumetric averaging method is employed to obtain the elastic properties of textile composites. The numerical simulations show that the fluctuations of geometric parameters may affect the elastic response of textile composites considered. The example of single straight yarn proves the validity of the theory and presents the explicit formulas of influence of stochastic yarn geometry on elastic constants. Within the example of plain weave composite, random yarn cross-section shape and twist degrade all of the elastic constants except the shear moduli and Poisson's ratios in 1–3 and 2–3 directions, which are determined by the comprehensive effect of the stochastic scaling and twist.

1. Introduction

With the textile composites increasingly being used in aeronautical areas, there are more and more studies in mechanical properties of textile composites. In the process of manufacturing of textile composites, it is a real physical phenomenon that the yarns (or tows) of textile have fluctuations with path and cross-section shape. The fluctuations of geometric parameters of yarns can lead to a scatter of the mechanical properties of composite.

Many authors and teams investigated the statistical features of the loci and internal geometry of yarns of textile, and reproduced virtual specimens by Markov Chain algorithm [1–6]. Bale et al. [1] analyzed statistically the shape and positioning of yarns in the 3-D woven composites by μCT. The information, including the yarn centroids and the area, aspect ratio, and orientation of the yarn cross-sections, are all analyzed. Blacklock et al. [2] developed the Markov Chain algorithm for generating replicas of textile composite with the same statistical characteristics as specimens imaged using tomography. Meanwhile, Rinaldi and coworkers [3] used analogous algorithms to generate 3-D yarn representations. The topological rules were defined to provide instructions for resolving interpenetrations or ordering errors among yarns. Compared to the energy minimization method [4], such a method adjusted interference between yarns and was proved to be an efficient geometric method. Similar to Ref. [1], Vanaerschot et al. [5] used the reference period collation method to laminated polymer composites and analyzed the systematic trends and stochastic deviations. Correlation lengths were also analyzed. Then they used WiseTex software to reproduce stochastic models and calibrate Markov Chain algorithm for textile fabrics[6].

Whitney [7] studied the effect of fiber shape on in-plane and bending properties of laminates by micromechanics model. Lee et al. [8] established analytic model to predict the geometric characteristics and the elastic constants of plain woven composites. Parametric study was conducted to investigate the effects of yarn crimp angle, the shape of yarn cross-section, and yarn size et al. on the elastic properties of plain woven composites, too. Yushanov and Bogdanovich [9,10] developed a general theory of the elastic constants of composite with random waviness of the reinforcements. Three types of composites including unidirectional, biaxial, and 3-D orthogonally woven reinforcements were analyzed. Fang et al. [11] used the stochastic theory [9] to analyze the influence of distorted yarns on elastic properties and strength of 3-D four directional braided composites.

However, the research regarding the effect of stochastic geometric shape of composite reinforcement on elastic constants is relatively few. Olave et al. [12] employed Monte Carlo simulation and WiseTex software to analyze the influence of meso-scale geometrical variability on laminate stiffness. The conclusions

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showed the laminate thickness and orientation are the largest contributors to the stiffness dispersion. Vanaerschot et al. [13] evaluated the effect of the geometrical variability at meso-scale on the mechanical behavior of textiles, and pointed out that, for 2/2 twill woven textile, the resulting stiffness variation was limited to less than 1% covariance with a mean value which was 3% lower than the nominal model constructed with the systematic patterns.

The purpose of this research is to establish a general theory of the influence of stochastic yarn geometric parameters on the elastic constants of woven composites. Firstly, a classification of the yarns in a textile composite is introduced in the paper. A general textile composite, included multi-layers composite, is divided into several yarn genera, which is a term defined in such a way that the yarns within a genus are statistically equivalent to one another in the repeating structure of the textile composite [3]. Each genus contains all of the yarns which have the same yarn direction and statistical properties of geometric parameters in the whole textile composite. The individual yarn of the genus is identified by path vector and every stochastic geometric parameter. Therefore, we can take an arbitrary yarn in a genus for example to introduce the means and statistical properties of geometric parameters of this genus into the compliance. A yarn with random cross-section shape is mapped to an ideal yarn by the variable metric coordinate transformation which converts an anisotropic problem induced by random shape of yarn into a metric transformation. Compared to existing approaches, one advantage of the developed theory is not required the application of anisotropic mechanics theory, but needs only the means and standard deviations of stochastic scaling factors and twist angle of each genus.

This paper is structured as follows: In Section 2, six parameters are defined to fully describe the yarn of textile composites (the first three of them, constituting path vector, denotes the loci of yarns; the others are geometry parameters which are defined to describe the shape of yarn cross-section); Section 3 described the evaluations of covariance of geometric parameters; Then, in Section 4, the expansion, mean and covariance of local stochastic variable metric basis (LSVMB) are evaluated in sequence; Finally, the mean of yarn compliance and global compliance averaging of textiles are calculated in Section 5.

2. Definitions of path and geometric parameters

In order to describe a yarn (or tow) of textile completely, it needs two parts of information: spatial position and geometry information. An arbitrary yarn in 3D space is considered, see Fig. 1. A set of orthogonal axes 1–3 is defined as global coordinate system where 1-axis is along the longitudinal direction of yarn. Three center position parameters (x_1, x_2, x_3) of yarn make up the path vector $\mathbf{r}$ to identify spatial position of yarn. The cross-section shape of yarn, normal to the path at any position, can be fitted to ellipse approximately. Another three geometric parameters (a, b, θ) are used to characterize geometry of yarn cross-section. A scaling matrix (detailed in Section 2.3) can be formed by the first two parameters (a, b) corresponding to semi-major and semi-minor axes of elliptic cross-section, respectively; the last parameter θ is the orientation of yarn's cross-section, which is a description of twist extent of yarn around the tangent vector of the yarn path. A triad of unit vectors $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is an orthogonal basis of the global coordinate system 1–2–3.

2.1. Path vector

Consider an arbitrary center path curve P_0P_1 of yarn, see Fig. 1. The path vector $\mathbf{r}$ is specified as a parametric form

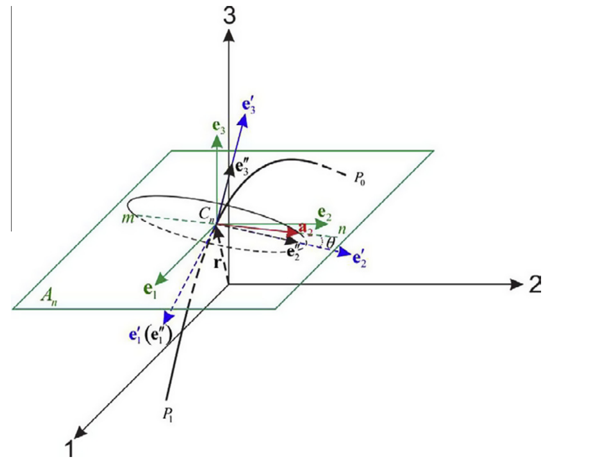

Fig. 1. Spatial path vector $\mathbf{r}$, twist angle θ, and global $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, local $\{\mathbf{e}_1', \mathbf{e}_2', \mathbf{e}_3'\}$, and variable metric basis $\{\mathbf{e}_1'', \mathbf{e}_2'', \mathbf{e}_3''\}$ related to the yarn path P_0P_1.

$$\mathbf{r}(\xi) = x_i(\xi)\mathbf{e}_i \tag{1}$$

where $\mathbf{e}_i (i = 1, 2, 3)$ are global basis, ξ is the running parameter in 1-axis direction.

The path vector $\mathbf{r}$ can be orthogonal decomposed as $x_2(\xi),\ x_3(\xi)$ in 1–2 and 1–3 planes, respectively, as depicted in Fig. 2. Accordingly, $x_2(\xi), x_3(\xi)$ define path curve of yarn in 1–2 and 1–3 planes respectively.

2.2. Twist angle

A triad of unit vectors $\{\mathbf{e}_i'\}$ at any point along the path forms a local orthogonal basis of local coordinate system 1'–2'–3'. Consider a yarn cross-section at an arbitrary point C_n on the yarn path P_0P_1, see Fig. 1. If unit vector $\mathbf{e}_1'$ is selected as tangent vector of P_0P_1, the unit vectors $\mathbf{e}_2',\ \mathbf{e}_3'$ are chosen as semi-major and semi-minor axis of yarn cross-section, respectively. Plane A_n is parallel to 1–2 plane at point C_n, and mn is the line of intersection of A_n and yarn cross-section at point C_n. The twist angle θ is defined as the angle between the semi-major axis of yarn cross-section and the intersecting line mn, which can be decomposed into two parts:

$$\theta = \theta(\xi) = \langle \theta(\xi) \rangle + \mathring{\theta}(\xi) \tag{2}$$

where $\langle \theta(\xi) \rangle$ is the mean of twist angle, and $\mathring{\theta}(\xi)$ is stochastic centered function.

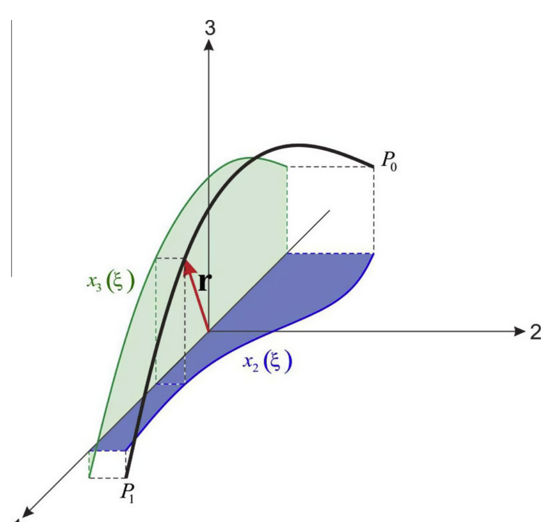

Fig. 2. The orthogonal decomposition of path vector $\mathbf{r}$ in 1–2 and 1–3 planes.