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# Prediction of uncertain elastic parameters of a braided composite

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ABSTRACT

Uncertainties are generally included in elastic parameters of the fiber reinforced textile composites. An approach on predicting the statistical characteristics of elastic modulus of braided composites using vibration test data is proposed in this paper. The method consists of two main steps, firstly estimate the initial mean values of the orthotropic elastic parameters by adopting the homogenization method, and then identify the variability of the elastic constants using a stochastic model updating algorithm with the first-order perturbation approach, the uncertain parameters are assumed as the summation of mean value and a deviation term with zero mean. A braided composite panel is employed in numerical simulations to verify the proposed method, the random samples of experimental modal data are simulated with the Latin Hypercube Sampling technique based on a reference finite element model, which is constructed by using deterministic updated elastic parameters. Results show that the mean value and standard deviation of the uncertain effective parameters of braided composite can be identified from modal data using the proposed stochastic identification method.



## 1. Introduction

Fiber braided composite materials have been widely applied in engineering due to the outstanding properties, such as high stiffness and strength to weight ratio, good fatigue strength, and excellent thermal protective characteristics. Since elastic properties are very important mechanical parameters of composite materials, prediction of which has become an active research area [1,2].

Uncertainties are generally included in the elastic modulus because of the complexities of textile composites, such as the properties of fibers and matrix are indeterminate, fiber shapes and the array model of fibers are irregular, porosities and micro-cracks caused by thermal residual stress [3] distribute in the matrix or fibers [4], and so on. Therefore, variability in elastic constants is necessary to be considered in mechanical response analysis of composite structures [5,6].

Investigations on predicting the equivalent elastic modulus of fiber braided composites have been extensively undertaken and most of the approaches [6,7] are deterministic, including methods based on experimental testing, theoretical analysis and numerical simulation [8–10]. Generally, elastic properties of composites can be obtained accurately via experimental tests, but these methods are usually expensive and time-consuming, moreover, it is difficult to obtain the whole orthotropic stiffness tensor including nine independent coefficients. However, both the analytical methods and numerical methods based on homogenization models of unit cell or representative volume element (RVE) of the composite [11] can be treated as alternative solutions to evaluate the equivalent elastic modulus. Numerous studies on the analytical and numerical methods [1,2,12–14] have been conducted. Dixit and Mali [13] concluded that the finite element analysis of unit cell in association with the homogenization technique is of crucial importance in predicting the effective elastic properties. Cox and Flanagan [15] described the basic ideas of a semi-analytic models to predict the elastic parameters of the composite material. Two simple and convenient analytical models for calculating elastic properties of woven fabric composites was also proposed by Bystrom and Jekabsons [16]. Wang et al. [17] adopted the numerical approach to predict the effective modulus of 2D C/SiC composite in which the voids and micro-cracks are generated in the chemical vapor infiltration process were considered.

In the process of equal unit cell or representative volume element of composite materials to be a homogenized model, the indispensable idealization assumptions will inevitably introduce errors into the equivalent predictions. Hallal et al. [18] proposed an improved analytical modeling method for a 2.5D interlock

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woven composite, and indicated that a model just based on the iso-strain assumption could not give accurate prediction of the equivalent elastic modulus. By comparing the numerical predictions with the results from tensile test, Pochiraju and Chou [19] indicated that the predicted deviations of elastic properties are largely within 10–15% of the experimental values for braided composites. Dalmaz and Ducret et al. [8] measured seven of the nine independent elastic constants of a woven 2.5D carbon-fiber reinforced SiC ceramic matrix by an ultrasonic technique associated with a numerical optimization process, it is reported that uncertainties are included in the seven obtained stiffness matrix components $C_{ij}$, and variations up to 20% can be found on the Young's Modulus, 25% on the Poisson's ratio. Therefore, elastic modulus predicted by analytical and numerical methods have to be calibrated by the results from repeated experimental tests.

Uncertainties in determining the equivalent elastic properties can be classified into three categories, (i) numerical uncertainties (truncation errors, discretization errors...), (ii) parametric uncertainties (properties of fibers and matrix, boundary conditions...), (iii) experimental uncertainties (experiment condition, noise, modal identification method...). The three kinds of uncertainties will ultimately result in difficulties in predicting the discreteness of elastic modulus. Up to now, limited attention has been focused towards identifying the variability of equivalent elastic modulus in the textile composite, and few attempts have been made to determine the uncertain stiffness tensor of this kind of composite based on parameter identification method using the experimental data of macro-mechanical properties, especially the modal data which can be measured easily from vibration tests.

Deterministic model updating technique is an inverse procedure using modal frequencies, mode-shapes, or frequency response functions to estimate updating parameters, which is a useful tool for adjusting mechanical parameters in finite element models. This method was described in detail by Friswell and Mottershead [20,21] and in recent years has been developed into a mature technology and applied successfully in engineering. On the premise of initial finite element model with good understanding of the structure and the mechanical properties [22], model updating can be used for identifying parameters of structures effectively [23]. When uncertainties exist in elastic properties of C/SiC composite, randomness of parameters can also be identified by using the stochastic model updating approach, which is developed based on the integration of uncertainty propagation approaches [24–27] with deterministic model updating algorithms. For different types of uncertainties, different kinds of model updating methods involving variations have been investigated, such as Monte-Carlo inverse procedure [28], perturbation methods [29,30], maximum likelihood method [31], Bayesian statistical framework [32], interval [33] and inverse fuzzy arithmetic approaches [34], and methods based on treating uncertainties as spatially correlated random fields [35]. The perturbation based model updating algorithm is adopted to identify the variability of elastic parameters, because of which is promising and with good computational efficiency for applying in engineering.

In this study, a methodology is proposed to identify the uncertain elastic modulus of braided composites using modal data. The outline of the work is as follows: the homogenization method for predicting equivalent elastic modules of periodic composite is introduced in Section 2.1. The basic theory of stochastic model updating based on perturbation method is addressed in Section 2.2. The deterministic model updating of a braided composite panel with experimental modal data will be presented in Section 3.1 and the reference finite element model is obtained. In section 3.2, Numerical simulations are conducted to verify the proposed stochastic parameter identification method.

## 2. Basic theory

### 2.1. Equivalent elastic modules

The so called layer-to-layer angle-interlock woven composite as shown in Fig. 1 is under investigated. In order to obtain the initial mean values of the elastic properties of the composite for identification, a homogenization problem is formulated with a unit cell. Predictions of the mechanical properties are significantly relying on two geometric parameters, cross-sectional shape of yarns and curved line of the warp weaving path. The corresponding geometric hypothesis are: (1) the cross-sectional shapes of the weft weavers and warp yarns are supposed respectively to be ellipsoidal and rectangular; (2) the warp weaving path is simulated by a curve and a tangent straight line; under these assumptions the finite element model of the unit cell can be constructed. Consequently, the macroscopic stiffness matrix **C** and the macroscopic compliance matrix **S** will be obtained by using Iso-strain or Iso-stress method [1].

Generalized Hook's law for an elastic anisotropic material is a linear stress–strain constitutive relationship defined by

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} \tag{1}$$

$$\boldsymbol{\varepsilon} = \mathbf{S}\boldsymbol{\sigma} \tag{2}$$

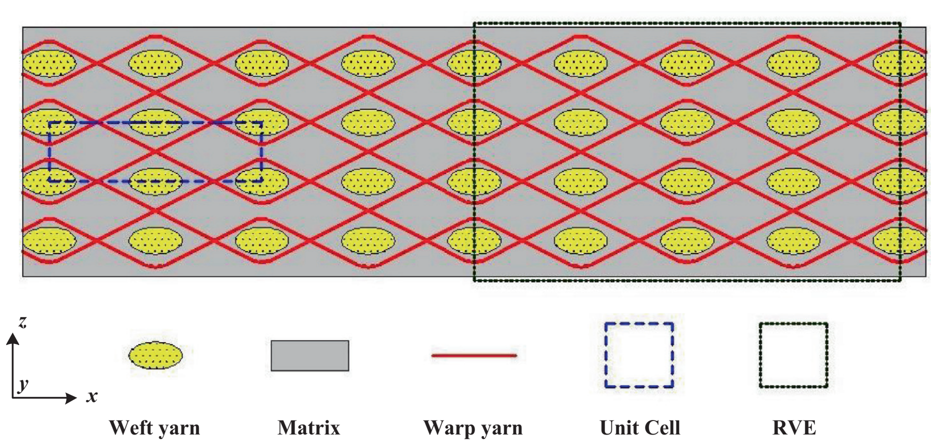


**Fig. 1.** Structure of 2.5D braided composite material.