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Thermoelastic behavior of advanced composite sandwich plates by using a new 6 unknown quasi-3D hybrid type HSDT

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ABSTRACT

This paper presents an analytical solution for the thermoelastic bending analysis of advanced composite sandwich plates by using a new quasi-3D hybrid type HSDT with 6 unknowns which is based on a generalized formulation. In addition, the nonlinear term of the temperature field is included in the generalized mathematical formulation in such way that it can be freely chosen and if desired can be different from the shear strain shape functions of the displacement field. So, infinite quasi-3D hybrid type HSDTs with just 6 unknowns can be derived from the present generalized formulation. The thermoelastic bending governing equations are obtained through the principle of virtual works and solved via Navier Method. Interesting results are obtained and compared with quasi-3D and 2D HSDTs. Transverse shear stress results are strongly influenced by nonlinear temperature field and for different HSDTs different results are produced. Therefore should be further discussed in the literature.

1. Introduction

Functionally graded materials (FGMs) are a type of heterogeneous composite material in which the properties change gradually over one or more directions. This material is produced by mixing two or more materials in a certain volume ratio (commonly ceramic and metal). FGMs have been proposed [1], developed and successfully used in industrial applications since 1980's [2]. Its counterpart, classical composites structures such as fiber reinforced plastics (FRPs) suffer from discontinuity of material properties at the interface of the layers and constituents. The continuous nature of the variation of the material properties in FGMs lessens the stress concentrations which become troublesome in a classical composites structure. FGMs were initially designed as a thermal barrier for aerospace structures and fusion reactors. They are now being developed for general use as structural components subjected to high temperatures. Nowadays, FGMs are an alternative materials widely used in aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical and shipbuilding industries.

On other hand, many shear deformation theories have been developed over the last years for the analysis of structural elements. These theories can be divided in two groups by a simple criterion: shear deformation theories with thickness stretching effect and shear deformation theories without thickness stretching effect. When a theory includes the thickness stretching effect, the transverse displacement is considered dependent by thickness coordinates obeying the Koiter's recommendation [3], i.e., $\varepsilon_{zz} \neq 0$. In the literature many theories that include thickness stretching effect to study the static, dynamic and stability behavior of functionally graded (FG) single-layer and sandwich plates subjected to thermal or/and mechanical load can be found. Below are listed the most relevant works:

Zenkour [4] investigated the static problem of exponentially graded (EG) rectangular plates subjected to transverse mechanical load using both quasi-3D trigonometric plate theory (TPT) and 3D elasticity solution. The quasi-3D TPT presented in this paper includes the thickness stretching effect, $\varepsilon_{zz} \neq 0$. The thermoelastic bending problem of FG sandwich plates (consisting of a homogeneous core with two FG face-sheet layers) were studied by Zenkour and Alghamdi [5], using a HSDT with thickness stretching effect. The authors modeled nonlinear temperature distribution with a sine function.

Matsunaga [6] modeled the displacement field with power series of the thickness coordinate for the analysis of functionally graded plates (FGPs) under thermal and mechanical loads based on 2D HSDT. Carrera et al. [7] studied the effects of thickness

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stretching in FGPs and shells under mechanical loads. For analysis, the authors propose several shear deformation theories by using the Carrera's Unified Formulation (CUF). The importance of the transverse normal strain effects in mechanical prediction of stresses for FGPs was pointed out. Neves et al. [8,9] presented a quasi-3D sinusoidal and hyperbolic shear deformation theory ($\varepsilon_{zz} \neq 0$), respectively, for the static and free vibration analysis of FGPs by using collocation with radial basis functions. Mantari and Guedes Soares [10–12] developed new quasi-3D HSDTs ($\varepsilon_{zz} \neq 0$) for the study of the static analysis of FGPs. In [10,12] the authors presented generalized formulations for the displacement field with two shape functions that are independent of each other. Houari et al. [13] analyzed the sandwich plates with FG skins under thermal load by using a trigonometric HSDT with thickness stretching effect. Readers can also consult the non-polynomial HSDTs presented in Refs. [14–17]. For example, in Saidi et al. [14], in addition to the thermoelasticity analysis of FGMs, as in this paper, they also studied the thermomechanical effect on FGPs.

Many authors use non-polynomial shear strain shape functions, such as trigonometric, trigonometric hyperbolic, exponential, etc. However, the thickness expansion modeling is conditioned by the in-plane displacement model (the transverse non-linear function in the modeling of the thickness expansion is an even function which usually is the derivative of the in-plane non-linear shear strain shape function, i.e. $g(z) = f'(z)$). Therefore, it is not free to choose a different shear strain shape function of the thickness expansion. The present formulation has that freedom, and infinite quasi-3D hybrid type shear deformation theories (polynomial or non-polynomial or hybrid type) can be created just having six unknowns.

In the present paper, a generalized formulation for the thermoelastic bending analysis of FG sandwich plates is presented. This generalized quasi-3D hybrid type HSDT accounts for adequate distribution of the transverse shear stresses through the plate thickness and tangential stress-free boundary conditions on the plate boundary surface, thus a shear correction factor is not required. The nonlinear term of the temperature field can be different from the shape functions of the displacement field, i.e. is also formulated in generalized manner. The mechanical properties of functionally graded layers of the plate are assumed to vary in the thickness direction according to a power law distribution in terms of the volume fractions of the constituents. The governing equations for the thermoelastic static analysis are obtained through the principle of virtual work. Navier-type analytical solutions are obtained for FG sandwich plates subjected to transverse thermal bi-sinusoidal load for simply supported boundary conditions. The performance of this theory is verified by comparing it with other quasi-3D and 2D HSDTs available in literature. Transverse shear stress results are strongly influenced by nonlinear temperature field and for different HSDTs different results are produced. Therefore should be further discussed in the literature.

2. Theoretical formulation

The sandwich plates of uniform thickness "h", length "a", and width "b" is shown in Fig. 1. The rectangular Cartesian coordinate system x, y, z, has the plane $z = 0$, coinciding with the mid-surface of the plate. The vertical positions of bottom, the two interfaces and the top surface of the sandwich plate are denoted by $h_1 = -h/2, h_2, h_3, h_4 = h/2$, respectively. The ratio of the thickness of each layers from bottom to top is denoted by the combination of three numbers, for example, a symmetric sandwich plate composed of three layers of equal thickness will have a configuration or scheme "1-1-1" ($h_2 = -h/6, h_3 = h/6$).

2.1. Functionally graded sandwich plates

The material properties for the functionally graded layers vary through the thickness with a power law distribution, which is given below:

$$P^{(k)}_{(z)} = \left(P^{(k)}_t - P^{(k)}_b\right) V^{(k)}_{(z)} + P^{(k)}_b, \tag{1}$$

where $P^{(k)}$ denotes the effective material property, $P^{(k)}_t$ and $P^{(k)}_b$ denote the property of the top and bottom faces of the functionally graded layer, respectively, and "k" represent a single-layer of the sandwich plate, i.e., $k = 1, 2, 3$ for the bottom, middle and top layer, respectively. The effective material properties of the plate, including Young's modulus, E, and shear modulus, G, and the thermal expansion coefficients, "α", vary according to Eq. (1). Generally, Poisson's ratio, "ν", varies in a small range. For simplicity, in this paper, "ν" is assumed constant (see Ref. [14]).

The sandwich plate is composed of three layers, an isotropic core and two functionally graded skins as shown in Fig. 1. The core is a fully ceramic layer, the bottom layer is made of a mixture of materials from metal to ceramic and the top layer is made of a mixture of materials from ceramic to metal. Therefore, the volume fraction for the ceramic phase $V^{(k)}$ is expressed as (see Fig. 2):

$$\begin{aligned}
V^{(1)}_{(z)} &= \left(\frac{z - h_1}{h_2 - h_1}\right)^p, \quad h_1 \leqslant z \leqslant h_2,\\
V^{(2)}_{(z)} &= 1, \quad h_2 \leqslant z \leqslant h_3,\\
V^{(3)}_{(z)} &= \left(\frac{z - h_4}{h_3 - h_4}\right)^p, \quad h_3 \leqslant z \leqslant h_4,
\end{aligned} \tag{2a – c}$$

where "p" is the exponent that specifies the material variation profile through the thickness ($0 \leqslant p \leqslant \infty$).

From the above equations can be stated that if the exponent is equal to zero ($p = 0$), the layer acquires the material properties of the top surface. Likewise, if the exponent is equal to infinity ($p = \infty$), the layer acquires the material properties of the bottom surface. These considerations are important when studying a homogeneous material.

2.2. Displacement base field

The generalized displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point $(x, y, \pm h/2)$ on the outer (top) and inner (bottom) surfaces of the plate, is given as follows (see Mantari and Guedes Soares [10]):

$$\begin{aligned}
\bar{u}(x,y,z) &= u(x,y) + z\left[y^*\theta_1 + q^*\tfrac{\partial\theta_3}{\partial x} - \tfrac{\partial w}{\partial x}\right] + f(z)\theta_1,\\
\bar{v}(x,y,z) &= v(x,y) + z\left[y^*\theta_2 + q^*\tfrac{\partial\theta_3}{\partial y} - \tfrac{\partial w}{\partial y}\right] + f(z)\theta_2,\\
\bar{w}(x,y,z) &= w + g(z)\theta_3,
\end{aligned} \tag{3a – c}$$

where $u(x,y), v(x,y), w(x,y), \theta_1(x,y), \theta_2(x,y)$ and $\theta_3(x,y)$ are the six unknown displacement functions of middle surface of the plate,

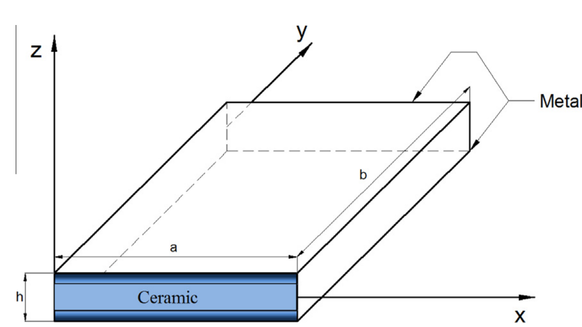

Fig. 1. Geometry of functionally graded sandwich plate.