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# Nonlinear reduced order modeling of functionally graded plates subjected to random load in thermal environment

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ABSTRACT

In this paper, large amplitude vibration of functionally graded material (FGM) plates subjected to combined random pressure and thermal load is studied using finite element modal reduction method. The material properties, which are depended on the temperature, vary in the thickness direction by a simple power law distribution in terms of the volume fraction of the constituents. The equations of motion in structural node degrees of freedom (DOF) are obtained based on von-Karman large deflection and first order shear deformation theory. The order of these equations is reduced using a novel approach for selection of the base vectors. Then the numerical results of the obtained reduced-order equations are compared with those of reduced-order equations obtained by other base vectors and also with those of full finite element method. It is shown that the proposed set of base vectors forms an excellent candidate for reducing the order of the equations of motion.



## 1. Introduction

Sonic fatigue has been recognized as an important problem in the aerospace technology. Aircraft panels can fatigue under strong acoustical loads in thermal environment. Therefore, accurate calculation of stress fields to predict the fatigue life is very important. When aircraft panels are subjected to strong acoustical excitation in high temperature, the stress-displacement relations are nonlinear. Therefore, the governing nonlinear partial differential equations of motion of such systems with complex geometry and boundary/initial conditions can only be approximately solved by numerical methods such as finite element method and Newmark approaches for spatial and temporal discretization, respectively. To reduce the computational cost and time effort especially in model based controlling of such systems, the order of the resulting large system of ordinary differential equations obtained by for example finite element method can be reduced using mode summation approaches. For more accurate prediction of the system response, the order of the equations of motion should be reduced using a multi-mode approximation instead of a single mode one. The selection of base vectors and determination of nonlinear stiffness coefficients in the reduced-order equations are difficult tasks even for simple structures. There are two approaches for calculating the nonlinear stiffness coefficients: direct approach and indirect approach. In the direct approach, presented by Nash [1] and Shi and Mei [2] these coefficients are calculated using a special finite element code. This approach is effective and accurate. In the indirect approach, presented by McEwan [3] and Muravyov and Rizzi [4,5] the nonlinear stiffness coefficients are calculated using static solutions of a fully nonlinear finite element model of the system. Although in this approach the coefficients are not meticulous but they can be easily calculated even for complex structures. In order to select the base vectors, almost all researchers used linear normal modes. Recently, combinations of linear normal modes with other base vectors presenting the effect of nonlinearity are suggested. For considering the in-plane displacement effects, different approaches such as normal membrane modes (NMM), normal modes (dominant transverse modes) combined with dual modes (TD), explicit physical condensation of membrane degrees of freedom (EPCM), explicit modal condensation and membrane effects (EMCM), implicit condensation of membrane (IC), implicit condensation and expansion of membrane effects (ICE), and system identification for basis selection (SIB) have been used. Rizzi and Przekop [6–8] suggested NMM method for predicting the in-plane motions. Although this method was successfully used by them for some applications, but the selection of normal membrane modes and their proper number is not an easy task, even for simple structures. Hollkamp et al. [9] compared these methods using the dynamic response of a simple clamed–clamped beam. They concluded that IC method is more practical than others. Przekop and Rizzi [10–14], Spottswood

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[15], Feeny [16,17], Chelidze [18] studied the dynamic response of systems using SIB method. The basic idea in SIB approach was formed by a snapshot matrix using the results of full finite element method. Then the columns of snapshot matrix are projected on the linear normal modes. The normal modes that have greater percentage in the response form the base vectors used for reducing the order of equations of motion. Gordon and Hollkamp [19] studied the dynamic response of a simple shallow beam using IC, ICE and full finite element methods. They demonstrated the usefulness of ICE method. Gorden and Hollkamp [20] compared the results of aforementioned methods with experimental results for a clamped–clamped beam. Mignolet [21], Radu [22] and Hollkamp [23] used TD method to represent the in-plane motions. They used two simple nonlinear static responses for construction of these dual modes. Kim et al. [24] presented another approach to construct new dual modes (KTD). They showed the efficiency and practically of these new dual modes for many complex structures. Mignolent et al., [25] reviewed some of the direct and indirect methods and showed that ICE, KTD and SIB methods were successful to predict the response of complex structures. In the study of dynamic thermal buckling response of structures, Javaheri [26] presented the equilibrium and stability of FGM plates under thermal loads using classical plate theory. Guo et al., [27] studied the dynamic thermal buckling response of laminated composite shallow shells using EPCM. They also obtained thermal buckling branches and showed two stable equilibrium configurations. Perez et al. [28] validated KTD method by studying the dynamic thermal buckling response of a FGM plate around its stable equilibrium positions. Ibrahim et al. [29] presented limit cycle oscillations and post buckling deflection of FGM plates subjected to aerodynamic and thermal loads using EPCM method. Sha et al. [30] studied the snap through and fatigue life prediction of curved panels using a fully nonlinear finite element method. Alijani et al. [31] investigated the dynamic response of FGM plates under harmonic excitation in thermal environment. They added supplementary nonlinear terms to include the in-plane displacements. The dynamic thermal buckling response of FGM plates using three dimensional theory of elasticity and classical plate theory was studied by Dogan [32]. Allahverdizade et al. [33] presented the dynamic response of electrorheological (FGER) beams using EPCM method.

In this study a special nonlinear finite element code based on von-Karman large deflection and first order shear deformation theory is developed for large amplitude vibration of FGM plates subjected to random pressure in thermal environment. The order of the equations of motion in the structural node degrees of freedom are reduced using different approaches including dominant transverse modes (DTM), EPCM, KTD, SIB, and a novel approach developed in this study. This novel approach, which is a combination of KTD and SIB methods, is named as modified system identification base (MSIB) method in the present study. The results of the reduced-order equations obtained by the proposed method will be compared with those of other reduced-order methods and also full finite element method in three nonlinear regimes: small amplitude vibration around one of two stable equilibrium positions, snap through response, and persistent snap through response.

## 2. Formulation

### 2.1. Equations of motion in structure node DOF

In this study the material properties of the FGM plates are assumed to be functions of two variables: location and temperature. The material properties vary in the thickness direction based on a simple power law distribution in the following form [34,35].

$$P(z,T) = P_b(T) + (P_t(T) - P_b(T))\left(\frac{2z+h}{2h}\right)^n \tag{1}$$

where $P(z, T)$ is the effective material property. $P_t(T)$ and $P_b(T)$ are the material properties of the top and bottom surfaces of the FGM plate. $h$ and $n$ are the thickness of the FGM plate and the volume fraction, respectively. $T$ is temperature assumed uniformly distributed throughout the FGM plate. The equations of motions were obtained by Przekop [36] for laminated shell and plate based on the first order shear deformation theory and large-amplitude vibration using Mindlin plate elements (MIN3). The global system equations of motion can be expressed as [36].

$$\begin{bmatrix} M_b & 0 \\ 0 & M_m \end{bmatrix}\begin{Bmatrix} \ddot{W}_b \\ \ddot{W}_m \end{Bmatrix} + \left( \begin{bmatrix} K_b & K_{bm} \\ K_{mb} & K_m \end{bmatrix} + \begin{bmatrix} K_b^s & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} K_{\Delta T} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} K1_b & K1_{bm} \\ K1_{mb} & 0 \end{bmatrix} + \begin{bmatrix} K1_b^{N_b} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} K1_b^{N_m} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} K2_b & 0 \\ 0 & 0 \end{bmatrix} \right)\begin{Bmatrix} W_b \\ W_m \end{Bmatrix} = \begin{Bmatrix} f_b \\ 0 \end{Bmatrix} + \begin{Bmatrix} f_b^{\Delta T} \\ f_m^{\Delta T} \end{Bmatrix} \tag{2}$$

where $W_b$ and $W_m$ are nodal bending and membrane displacements, respectively. Other terms are defined in [37]. Eq. (2) can be written in the following compact form

$$[M]\left\{\ddot{W}\right\} + \left([K_L] - [K_{\Delta T}] + [K1(W)] + \left[K2(W^2)\right]\right)\{W\} = \{P_r\} + \{P_{\Delta T}\} \tag{3}$$

where $[M]$ is the mass matrix, $[K_L]$ contains first two linear parts of the stiffness matrix, $[K_{\Delta T}]$ is the third part of the stiffness matrix showing thermal effect, $[K1(W)]$ and $[K2(W^2)]$ presented the linear and quadratic parts of the nonlinear stiffness matrixes. $\{P_r\}$ and $\{P_{\Delta T}\}$ are the external random force vector and thermal load vector, respectively. Since Eq. (3) contains nonlinear terms (stiffness matrices) and random terms (external force), its solution is very time consuming and difficult.

### 2.2. Equations of motion in modal coordinates

The order of the equations of motion in structural node DOF can be reduced to modal coordinates using different types of base vectors. The plate deflection can be expressed as a linear combination of some known base vectors as

$$\{W\} = \sum_{i=1}^{N}\{\varphi_i\}q_i(t) = [\phi]\{q\} \tag{4}$$

where $N$ is the number of base vectors which can be smaller than the number of structure node DOF. There are several methods for selection of these base vectors. In the following, KTD, EPCM, and SIB methods are shortly described. More detail can be found in [6,10,20,21,24,25] and [37].

#### 2.2.1. Explicit physical condensation of membrane DOF (EPCM)

In the physical coordinates, the system of Eq. (2) can be separated in two equations in the following form

$$\begin{aligned}&[M_b]\{\ddot{W}_b\} \\ &+\left([K_b]+[K_b^s]-[K_{\Delta T}]+[K1_b]+[K1_b^{N_b}]+[K1_b^{N_m}]+[K2_b]\right)\{W_b\} \\ &+([K_{bm}]+[K1_{bm}])\{W_m\} \\ &=\{f_b\}+\{f_b^{\Delta T}\}\end{aligned} \tag{5}$$

$$[M_m]\{\ddot{W}_m\} + ([K_{mb}] + [K1_{mb}])\{W_b\} + ([K_m])\{W_m\} = \{f_m^{\Delta T}\} \tag{6}$$