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An anisotropic in-plane and out-of-plane elasto-plastic continuum model for paperboard

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ABSTRACT

A continuum model of paperboard material possessing a high degree of anisotropy is established. To handle the anisotropy, three vectors are introduced which phenomenologically represent the preferred directions of the material. The in-plane director vectors deform as line segments and the out-of-plane direction deforms as a normal vector. This allows for a decoupling of the in-plane and the out-of-plane responses in shearing. The model is developed for large plastic strains and consequently an expression for the plastic spin has been proposed. The choice of plastic spin allows for a control of the direction in which permanent deformations will occur. To show the predictive capabilities of the model, the important industrial process of creasing is simulated. Both the simplified line crease setup, as well as the actual rotation crease setup used in industrial applications are studied.

1. Introduction

Paperboard is a material with a high degree of anisotropy, which stems from the manufacturing process where the fibers becomes aligned in preferred directions, which results in a highly anisotropic structure. In this work, a 3D-continuum elasto-plastic model for paperboard is established, i.e. a model that captures the in-plane as well as the out-of-plane responses is developed. Classically, paperboard is characterized as an orthotropic material where the normals to the symmetry planes are denoted as Machine Direction (MD), Cross Direction (CD) and out-of-plane direction (ZD), cf. Fig. 1. The magnitude of the failure stress in the MD direction is typically 2–3 times higher compared with CD and about 100 times higher compared with the failure stress in the ZD-direction, cf. [34]. Different modeling concepts have traditionally been employed for the modeling of the in-plane and the out-of-plane responses, such as using a combination of continuum and cohesive elements, cf. [38,7,29]. In this work, a model, which is able to handle the large degree of anisotropy using a purely continuum based model, is presented.

Paperboard can be designed as a single-ply or multi-ply material, where the fibers in the plies are processed mechanically or chemically from wood fibers. The plies are designed to obtain desired properties through the thickness of the paperboard. The multi-ply board is a sandwiched structure, which is used to obtain a light weight construction with as high stiffness as possible without compromising other functionalities such as strength and convertibility. The multi-ply design utilizes strong outer plies with higher bending resistance to prevent cracks to form, while the middle plies are made weaker such that the material can easily be folded to form a package. If a single-ply board is used, a combination of chemical additives can be pressed into the top and bottom ply to obtain a layered structure. In this work however, focus will be on the modeling of materials with a high degree of anisotropy, and for that sake the inhomogeneous properties of paperboard in the thickness direction has not been taken into account. The inhomogenous material properties can easily be included in the framework by a mapping of the material properties, cf. [20]. The different through thickness shear properties can be identified by using a notched shear test, as developed in [28,30] or by grinding off the plies and testing the material properties of individual plies cf. [27]. Several mechanical characteristics for the in-plane behavior of paperboard were determined in [2], such as visco-elastic effects, plasticity and damage. Rate-dependence and damage have not been considered in the current work.

To obtain well formed packages without defects, creasing is an important industrial converting process and it is crucial for the subsequent folding operation. Creasing has been studied experimentally by several authors e.g. [11,12,26] and also in numerical studies in [7,21,29]. The creasing operation reduces the initial maximal bending moment and the deeper the scored line is

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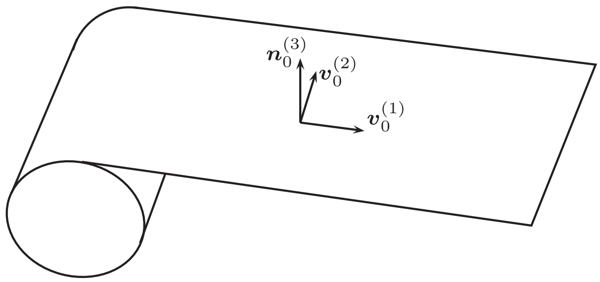

Fig. 1. Illustration of the different material directions of paperboard resulting the manufacturing process. The preferred directions are aligned with the Machine(1)-, Cross(2)- and ZD(3)-directions.

creased, the more the maximal bending moment is reduced. Earlier studies have modeled the simplified 2-dimensional line crease setup, whereas in this work a rotation crease with a 3-dimensional pattern is simulated as well. The modeling of the subsequent folding operation of paperboard has been investigated in [5,20,17].

There exist several modeling techniques for the modeling of fiberous structures such as paperboard. Network models have been proposed in e.g. [10,22] where the fiber network was built up by beam elements. Insights about the mechanics present at the meso- and micro-scale of the fiber-network can be obtained by using network models. To reduce the computional cost of network models, the quasicontinuum approach was applied to fiberous materials such as paperboard in [6]. Continuum models for paperboard have previously been suggested, e.g. the in-plane models defined in [32,23]. Combined continuum and delamination models have been proposed in [7,29,21,38].

The in-plane yield surface in [39] and further developed in [9] has been extended in the present work to take the out-of-plane properties into account. The yield surface is based on a set of subsurfaces in the stress-space, where each subsurface is associated with an internal variable. This approach allows the yield surface to harden distortionally in the stress-space. The yield surface in this work is equipped with additional subsurfaces such that out-of-plane plasticity is accounted for. It was observed in [37,35], that dilation in the ZD-direction is obtained as the paperboard is sheared, and that increased shear yield stress is obtained as the material is compressed. This feature is included in the proposed model. Ideal plasticity will be assumed at the onset of failure in the out-of-plane direction.

The article is organized as follows, in Sections 2 and 3 the kinematic description and the evolving anisotropy is presented. The thermodynamic framework is established in Section 4, where tensors will be considered in a Cartesian setting, i.e. following the work of [14]. In Section 5, the specific model is presented and in Section 6, aspects related to the calibration are discussed. The model is implemented in a finite element framework and the results from creasing operations are shown in Sections 7 and 8.

2. Kinematics

The motion of a material body from the reference configuration, $\Omega_0 \in \mathbb{R}^3$, to the current configuration $\Omega \in \mathbb{R}^3$ in the time interval $T \in [t_0, t]$, is given by $\boldsymbol{\varphi}(\boldsymbol{X}, t) : \Omega_0 \times T \rightarrow \Omega$. It is assumed that the mapping $\boldsymbol{\varphi}$ is sufficiently smooth. The vector $\boldsymbol{X}$ denotes the position of a particle in the reference configuration and the position of the same particle at time t in the current configuration is given by $\boldsymbol{x} = \boldsymbol{\varphi}(\boldsymbol{X}, t)$. The mapping of vectors in the reference configuration to the current configuration is given by the deformation gradient $\boldsymbol{F} = \nabla\boldsymbol{\varphi}$. To separate the deformation into an elastic and a plastic deformation, a multiplicative split of the deformation gradient is assumed, i.e.

$$\boldsymbol{F} = \boldsymbol{F}^e\boldsymbol{F}^p, \tag{1}$$

where $\boldsymbol{F}^e$ and $\boldsymbol{F}^p$ are the elastic and plastic deformation gradients, respectively. The split (1) introduces a stress-free intermediate configuration, which is not unique. A rigid body rotation of the intermediate configuration will leave the intermediate configuration stress free and therefore the intermediate configuration must be defined with respect to an arbitrary constitutive spin, cf. [16,19]. For simplicity in this paper, this constitutive spin is set equal to zero, i.e. an isoclinic configuration is adopted, as introduced in [24]. Further on, the elastic deformation will be defined by the elastic Finger tensor, $\boldsymbol{b}^e = \boldsymbol{F}^e(\boldsymbol{F}^e)^T$.

Using (1), the spatial velocity gradient defined as, $\boldsymbol{l} = \dot{\boldsymbol{F}}\boldsymbol{F}^{-1}$, can be additively split into

$$\boldsymbol{l} = \boldsymbol{l}^e + \boldsymbol{F}^e\boldsymbol{L}^p\boldsymbol{F}^{e-1} = \boldsymbol{l}^e + \boldsymbol{l}^p, \tag{2}$$

where

$$\boldsymbol{l}^e = \dot{\boldsymbol{F}}^e\boldsymbol{F}^{e-1}, \qquad \boldsymbol{L}^p = \dot{\boldsymbol{F}}^p\boldsymbol{F}^{p-1}, \tag{3}$$

are referred to as the elastic and material plastic velocity gradients, respectively. The plastic velocity gradient in (2) can further be split into a symmetric part and a skew-symmetric part, i.e.

$$\boldsymbol{l}^p = \mathrm{sym}(\boldsymbol{l}^p) + \mathrm{skew}(\boldsymbol{l}^p) = \boldsymbol{d}^p + \boldsymbol{\omega}^p, \tag{4}$$

where $\boldsymbol{d}^p$ is the plastic rate of deformation tensor and $\boldsymbol{\omega}^p$ is the Eulerian plastic spin, cf. [16]. The plastic spin, $\boldsymbol{\omega}^p$, is important to specify for anisotropic materials that undergo large plastic deformations, cf. [18]. For later purposes, the symmetric part of the spatial velocity gradient is defined as $\boldsymbol{d} = \mathrm{sym}(\boldsymbol{l})$.

3. Evolving anisotropy

The modeling framework for the anisotropy follows the format outlined in [9]. To model the in-plane behavior, two director vectors of unit length, $\boldsymbol{v}_0^{(1)}$ and $\boldsymbol{v}_0^{(2)}$, aligned in the MD- and CD-directions in the reference configuration are introduced. These two vectors are assumed to phenomenologically represent the in-plane preferred directions of the material. The director vectors are assumed to be embedded in the continuum (i.e. the fiber-network) and are chosen to follow the elastic deformation gradient i.e.

$$\begin{aligned} \boldsymbol{v}^{(1)} &= \boldsymbol{F}^e\boldsymbol{v}_0^{(1)} \\ \boldsymbol{v}^{(2)} &= \boldsymbol{F}^e\boldsymbol{v}_0^{(2)}. \end{aligned} \tag{5}$$

Note that due to the intermediate configuration being isoclinic, the director vectors in the intermediate configuration become equal to $\boldsymbol{v}_0^{(1)}$ and $\boldsymbol{v}_0^{(2)}$, i.e. an identity mapping between the director vectors in the reference configuration to the intermediate configuration. Rather than using $\boldsymbol{v}^{(3)} = \boldsymbol{F}^e\boldsymbol{v}_0^{(3)}$, a normal vector $\boldsymbol{n}_0^{(3)}$ will be utilized for the out-of-plane behavior. The normal vector $\boldsymbol{n}_0^{(3)}$ in the reference configuration is expressed as

$$\boldsymbol{n}_0^{(3)} = \boldsymbol{v}_0^{(1)} \times \boldsymbol{v}_0^{(2)}, \tag{6}$$

i.e. a vector normal to the in-plane directions. A normal vector evolve according to the cofactor of the elastic deformation gradient,

$$\boldsymbol{n}^{(3)} = J^e\boldsymbol{F}^{e-T}\boldsymbol{n}_0^{(3)}, \tag{7}$$

where J^e is the determinant of the elastic deformation gradient, i.e. $J^e = \det(\boldsymbol{F}^e)$. The use of (7) is motivated by the fact that paperboard in essence is a sandwiched structure, consisting of layers of fibers