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The effects of unequal compressive/tensile moduli of composites

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ABSTRACT

This paper investigates the effects of unequal compressive and tensile moduli of carbon fibre reinforced plastic (CFRP) composites. The basic assumption is based on the statistics that the compressive modulus is a fraction lower than the tensile modulus. Data evaluated by Finite Element Analysis (FEA) model, Classical Laminate Theory (CLT) model, and experiment are used to investigate these effects. The terms of compressive modulus are successfully introduced into the Tsai–Wu failure criterion for the production of failure envelops, into the Classical Beam Theory (CBT) and CLT for the investigation of flexural behaviour as well as the fibre microbuckling model for the analysis of compressive failure. The study shows that the failure criteria shift from stress domain to strain domain when the compressive modulus is considered, and the strain dominated failure criteria could generally provide more accurate prediction in composite material. Therefore it is proposed to apply strain dominated failure criteria for composite design, testing and certificate.

1. Introduction

The use of high strength, lightweight carbon fibre reinforced plastic (CFRP) composites in renewable energy devices is growing steadily due to their superior anti-corrosion properties and the long-term fatigue performance [1,2]. According to the UK Engineering Integrity Society [3], a record of 22% of the UK's electricity supply was generated by wind. In other EU countries such as Germany, Spain and Denmark the record is approximately double. For many commercial CFRP composites, the longitudinal tensile strength can be five times higher than stainless steel with only one-fifth of its density. Besides the benefit of weight savings, it is possible to construct a rather huge structure for the renewable energy devices, such as the next-generation turbine blade.

In practical composite structures, the composite materials are subjected to complicated loading conditions, such as bending, tension, compression and twisting. A recent report of 3D FEA analysis [4] has demonstrated that all of the six stress components (σ_i, τ_{ij}) contribute to the failure criterion of CFRP composites, particularly the initiation of failure in bending. However, most of the previous studies on composites are based on equal compressive/tensile moduli, which may lead to either overestimate or underestimate the composite strength. The effects of unequal compressive/tensile moduli on the failure criterion of composites have not been reported.

Due to the fibre misalignment and manufacturing defects, the compressive modulus of long fibre composites is reasonably not expected to be equal to the tensile modulus [5–9]. This becomes important in flexural behaviour because the composites are under both compression and tension. A laminate with unequal moduli may not behave symmetrically in bending, such as the stress and strain distributions through-thickness, even though the layup is symmetric. Therefore, for many classical theories, such as Classical Beam Theory (CBT) and Classical Laminate Theory (CLT), the compressive modulus should be introduced in order to eliminate the unequal terms.

Several papers have described work to modify CBT in the flexural test for fibre reinforced plastic composites. Chamis [10–12] used continuum mechanics to derive the formula of maximum deflection in three-point bending using unequal compressive and tensile moduli. Zhou and Davies [13,14] used statistical methods and assumed a higher compressive modulus to characterise the failure mechanics of thick glass woven roving/polyester laminates. Mujika et al. [15,16] used strain gauges to determine the compressive and tensile moduli of unidirectional laminates by measuring the compressive strain and tensile strain at the top and bottom surfaces of specimens in three-point and four-point bending. However, the effects of unequal moduli on the flexural properties and the failure strength of multi-directional filament laminate composites have not been well understood.

In the present work, the compressive modulus is assumed to be a fraction lower than the tensile modulus based on the statistics of current commercial CFRP composites. The effects of unequal

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Nomenclature

$[a],\ [b],\ [d]$ block matrices of $\begin{bmatrix} a & b \\ b & d \end{bmatrix}$ matrix (inversed $\begin{bmatrix} A & B \\ B & D \end{bmatrix}$ matrix)
$h_1,\ h_2$ height of tensile sheet and compressive sheet
$r_1,\ r_2$ long/short radius of ellipse
r radius of a single fibre
s offset of neutral plane to mid-plane
t thickness of lamina
$t_1,\ t_2$ thickness of tensile sheet and compressive sheet
$w,\ h$ width and height of laminate
$[A],\ [B],\ [D]$ block matrices of $\begin{bmatrix} A & B \\ B & D \end{bmatrix}$ matrix
E^{app} apparent flexural modulus
$E_1,\ E_2,\ E_3$ principal elastic moduli of lamina
$E_1^c,\ E_1^t$ longitudinal compressive and tensile moduli
F_{ij} operator of Tsai–Wu failure criterion in stress space
I moment of inertia
$M,\ M_x$ moment
$N_{x,y,xy},\ M_{x,y,xy}$ force and moment per unit length
$Q_{ij},\ \bar{Q}_{ij}$ extensional compliance matrix of unidirectional and off-axis lamina
$T_\varepsilon,\ T_\sigma$ transformation matrices of strain and stress
U_{ij} operator of Tsai–Wu failure criterion in strain space
V_f fibre volume fraction
γ_m shear strain of matrix
$\theta,\ \theta_1,\ \theta_2$ angle
κ curvature
π circumference ratio
λ_0 half-wavelength of fibres microbuckling
λ ratio of compressive modulus to tensile modulus
$(\sigma_1^t)_{ult},\ (\sigma_1^c)_{ult}$ ultimate longitudinal tensile and compressive strength of lamina
$(\sigma_2^t)_{ult},\ (\sigma_2^c)_{ult}$ ultimate transverse tensile and compressive strength of lamina
τ_{12}^{ult} ultimate in-plane shear strength of lamina

compressive/tensile moduli on composites are investigated: (a) the composite failure criterion, particularly Tsai–Wu failure criterion, (b) a modified CBT for the flexural properties of unidirectional laminate and its failure mechanisms, (c) a modified CLT for the flexural properties of multi-directional laminate, and (d) fibre micro-buckling. Three research approaches are used in parallel: (a) Finite Element Analysis (FEA) is employed to investigate the stress and strain distributions within the laminates for the identification of the maximum critical strains and stresses, (b) CLT is applied to extract the flexural modulus and strain/stress distributions of multi-directional laminate with different stacks, and (c) experiment is carried out to provide the sufficient evidence to support this study.

2. Background

Considering the loading condition and possible micro-scale structural defects in long fibre reinforced plastics composites, the compressive modulus is likely to be different from the tensile modulus. This will be more obvious in CFRP than GFRP composites since the diameter of carbon fibre is normally smaller than that of glass fibre. It is well-known that the smaller diameter of carbon fibre performs higher tensile strength. However, according to the Euler beam theory, a beam with smaller cross-section also tends to be unstable (buckling) which may lead to lower compressive strength. This is the dilemma in composite manufacturing.

In Table 1, there are ten commercial CFRP composites and their ratios of compressive/tensile moduli are very close. For these CFRP composites, the average ratio of compressive modulus to tensile modulus is around 0.9. In fact, with the increase of statistical specimens, the standard deviation decreases and the coefficient of variation has a tiny drop from 5.8% to 4.6%, as shown in Fig. 1. The actual value depends on the volume fraction of fibres and the manufacturing process. The ratios of compressive/tensile strengths are also included in the statistics, and the average value presents around 60%–70%.

For the convenient expression, a parameter is introduced to indicate the ratio of longitudinal compressive modulus to tensile modulus,

$$\lambda = \frac{E_1^c}{E_1^t} \tag{1}$$

Table 1
Longitudinal tensile/compressive moduli of CFRP composites and their strengths.

	E_1^t (GPa)	E_1^c (GPa)	$\frac{E_1^c}{E_1^t}$	$(\sigma_1^t)_{ult}$ (GPa)	$(\sigma_1^c)_{ult}$ (GPa)	$\frac{(\sigma_1^c)_{ult}}{(\sigma_1^t)_{ult}}$
Celion 12k/938	136	119	0.87	1.88	1.39	0.74
AS4 12k/3502	133	124	0.93	1.78	1.41	0.79
HITEX 33 6k/E7K8	125	118	0.94	2.16	1.44	0.67
AS4 12k/938	154	125	0.81	2.17	1.57	0.73
AS4/3501-6	135	123	0.91	2.01	1.45	0.72
T300 15k/976	135	129	0.95	1.45	1.30	0.89
AS4 12k/997	137	123	0.89	2.25	1.58	0.70
IM6 12k/APC-2	149	134	0.90	2.41	1.15	0.48
HTS40/977-2 [17]	140	112	0.80	2.52	1.40	0.56
Cytec/977-2 [18]	165	152	0.92	2.69	1.59	0.59
Avg.	141	126	0.89	2.13	1.43	0.69
SDs	12	11	0.05	0.37	0.14	0.12
Coeff var	8.4%	8.7%	5.8%	17.3%	9.5%	17.5%

Data source: Polymer matrix composites material handbook [19]. The values were measured at 75 °F (23 °C), and normalised to $V_f = 60\%$.

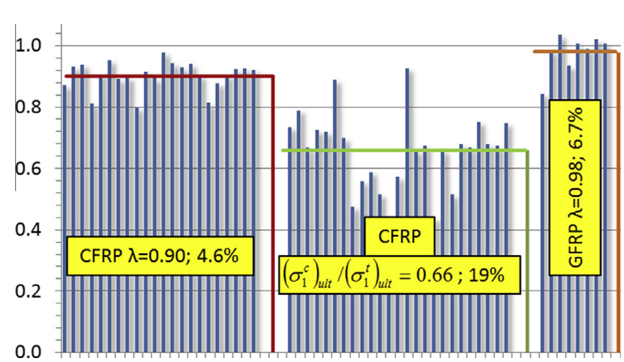

Fig. 1. Ratio of longitudinal compressive modulus to tensile modulus of various CFRP and GFRP. The average and their respective coefficient of variation are also shown in the figure.

Fig. 1 shows the λ value of various commercial CFRP and GFRP composites, and their coefficient of variation. The fibre volume fraction of CFRP and GFRP composites were normalised to