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Results on best theories for metallic and laminated shells including Layer-Wise models

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ABSTRACT

This paper deals with Best Theory Diagrams (BTDs) for metallic and laminated shells. The BTD is a curve that is defined over a 2D reference frame in which the horizontal axis indicates the error of a shell model with respect to a reference solution whereas the vertical axis indicates the number of displacement variables of the model. The best reduced model is a refined model that offers the lowest possible error for a given number of variables. The relevant terms of a model are detected by means of the axiomatic/asymptotic method (AAM), and the error is related to a given variable with respect to an exact or quasi-exact solution. In this work, a genetic algorithm has been used to obtain the BTD. The Carrera Unified Formulation (CUF) has been employed to build the refined models. The CUF makes it possible to generate automatically, and in a unified manner, any plate or shell models. Equivalent Single Layer (ESL) and Layer Wise (LW) refined models have been considered. The governing equations for shells have been obtained through the Principle of Virtual Displacements (PVD), and Navier-type closed form solutions have been considered. BTDs have been constructed by considering the influence of several parameters, such as various geometries, material properties, layouts, different displacement/stress components and loadings. The accuracies of some well-known theories have been evaluated and compared with BTD reduced models. The results suggest that, since the BTD depends on the problem characteristics to a great extent, the systematic adoption of the CUF and the AAM can be considered as a powerful tool to evaluate the accuracy of any structural theory against a reference solution for any structural problem.

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1. Introduction

Laminated composite and metallic shells are widely employed in several structural engineering applications, and many mathematical models have been developed over the last decades for the structural analysis of plates and shells. The solution of the 3D elasticity equations can be computationally prohibitive and valid only for a few geometries, material characteristics and boundary conditions. Computationally cheaper 2D structural models are commonly used to analyse shells and plates. The first model that was developed was the Kirchoff–Love ([1,2]). According to this model, transverse shear and normal strains are assumed to be negligible with respect to the other stress and strain components. An extension of this model to multilayered structures is referred to as the Classical Lamination Theory (CLT). Further details on shell theories can be found in [3].

The inclusion of shear effects can be carried out according to the Reissner–Mindlin model [4,5] that leads to the First Order Shear Deformation Theory (FSDT). Further refinements of the FSDT can be obtained through the Vlasov model and the Reddy–Vlasov model [6,7] to account for the homogeneous conditions for the transverse shear stresses at the top and bottom shell/plate surfaces.

A refined model that accounts for both the transverse shear and normal stress effects, i.e. that fulfills Koiter's recommendation [8], was developed by Hildebrand, Reissner and Thomas [9]. Other significant contributions on laminated shell models can be found in [10–17].

The number of unknown variables in the theories that were mentioned above is independent of the number of layers. These theories are commonly referred to as Equivalent Single Layer models (ESL). An alternative method is the Layer-Wise (LW) approach [18–23] in which each layer is seen as an independent plate and

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compatibility of displacement components are imposed at the interfaces. In an LW model, the number of unknown variables depends on the number of layers.

Multilayered structures are transversely anisotropic, and the mechanical properties are discontinuous along the thickness. These characteristics lead to transverse displacements whose slopes can rapidly change at the layer interfaces, and transversely discontinuous in-plane stresses. Due to equilibrium conditions, transverse stresses must be continuous at the interfaces. Zig-zag models [24,25] and mixed variational tools [26–28] have been developed over the last decades in which the displacement and stress fields can be assumed in each layer. Compatibility and equilibrium conditions are then used at the interfaces to reduce the number of the unknown variables.

The present paper deals with refined shell models that are built by means of the Carrera Unified Formulation (CUF) [29]. According to the CUF, the displacement field of shells and plates can be defined as an arbitrary expansion of the thickness coordinate. The expansion order is a free parameter of the analysis, and it can be chosen via a convergence analysis. The governing equations are obtained through a set of fundamental nuclei whose form does not depend on either the expansion order nor the choices made for the base functions.

In the CUF framework, a novel axiomatic/asymptotic method (AAM) has been recently developed for beams [30,31] and plates [32,33]. AAM allows us to investigate the effectiveness of each generalized displacement variable in detecting the solution for a given problem and different values of typical parameters such as the thickness, the orthotropic ratio and the stacking sequence. By retrieving only those variables that play a role in the detection of the mechanical behavior of the structure, this method leads to the definition of reduced models that have the same accuracy of the full model but that have fewer unknown variables. Another important outcome that stemmed from the use of the axiomatic/asymptotic approach was the Best Theory Diagram (BTD) [34]. The BTD is a diagram in which, for a given problem, the computationally cheapest structural model for a given accuracy can be read. The BTD is influenced by a number of geometrical and material parameters, and it can be obtained by exploiting genetic algorithms [35]. The most recent developments have dealt with the definition of more accurate techniques to evaluate the accuracy of the model [36,37] and with Layer-Wise plate models [38].

In this work, BTDs are obtained for metallic and laminated shells through genetic algorithms. Navier-like closed-form solutions are employed, and both ESL and LW models are considered. This paper is a companion work of [39] in which refined plate models were considered. This paper is organized as follows: the geometrical relations for shells and the constitutive equations for laminated structures are presented in Section 2; the CUF is presented in Section 3; the governing equations are introduced in Section 4; the axiomatic/asymptotic technique and the BTD are introduced in Section 5; the results are given in Section 6; the conclusions are drawn in Section 7.

2. Geometrical and constitutive relations for shells

This section deals with the basic geometrical and constitutive equations for multilayered shells. A more complete and detailed description of these equations can be found in [40]. The geometrical parameters and the reference frame of a generic k-layer of a multilayered shell are depicted in Fig. 1. The reference surface is defined as Ω_k, and its boundary is Γ_k. The reference system is defined by α_k, β_k, z, and the curvature radii along the principal directions are R^k_α and R^k_β. The following relation are valid in the given orthogonal system of curvilinear coordinates for the square of a line element, the area of an infinitesimal rectangle on Ω_k, and for an infinitesimal volume, respectively,

$$
\begin{aligned}
ds_k^2 &= H^k_\alpha d\alpha_k^2 + H^k_\beta d\beta_k^2 + H^k_z dz_k^2\\
d\Omega_k &= H^k_\alpha H^k_\beta \, d\alpha_k \, d\beta_k\\
dV_k &= H^k_\alpha H^k_\beta H^k_z \, d\alpha_k \, d\beta_k \, dz_k
\end{aligned}
\tag{1}
$$

where

$$
H^k_\alpha = A_k\left(1+\frac{z_k}{R^k_\alpha}\right) \qquad H^k_\beta = B_k\left(1+\frac{z_k}{R^k_\beta}\right) \qquad H^k_z = 1 \tag{2}
$$

Constant curvature shells were considered in this paper; that is, $A_k = B_k = 1$. For the sake of convenience, strain components were grouped into in-plane and out-of-plane components,

$$
\boldsymbol{\epsilon}^k_p = \begin{bmatrix}\epsilon_{\alpha\alpha} & \epsilon_{\beta\beta} & \epsilon_{\alpha\beta}\end{bmatrix} \qquad \boldsymbol{\epsilon}^k_n = \begin{bmatrix}\epsilon_{\alpha z} & \epsilon_{\beta z} & \epsilon_{zz}\end{bmatrix} \tag{3}
$$

and the strain–displacement relations were expressed as (see [3])

$$
\begin{aligned}
\boldsymbol{\epsilon}^k_p &= \mathbf{D}_p \mathbf{u}^k + \mathbf{A}_p \mathbf{u}^k\\
\boldsymbol{\epsilon}^k_n &= \mathbf{D}_n \mathbf{u}^k + \mathbf{A}_n \mathbf{u}^k = \mathbf{D}_{n\Omega} \mathbf{u}^k + \mathbf{D}_{nz} \mathbf{u}^k + \mathbf{A}_n \mathbf{u}^k
\end{aligned}
\tag{4}
$$

where

$$
\mathbf{D}_p = \begin{bmatrix} \frac{\partial_{\alpha_k}}{H^k_\alpha} & 0 & 0\\ 0 & \frac{\partial_{\beta k}}{H^k_\beta} & 0\\ \frac{\partial_{\beta k}}{H^k_\beta} & \frac{\partial_{\alpha_k}}{H^k_\alpha} & 0 \end{bmatrix} \qquad \mathbf{A}_p = \begin{bmatrix} 0 & 0 & \frac{1}{H^k_\alpha R^k_\alpha}\\ 0 & 0 & \frac{1}{H^k_\beta R^k_\beta}\\ 0 & 0 & 0 \end{bmatrix} \tag{5}
$$

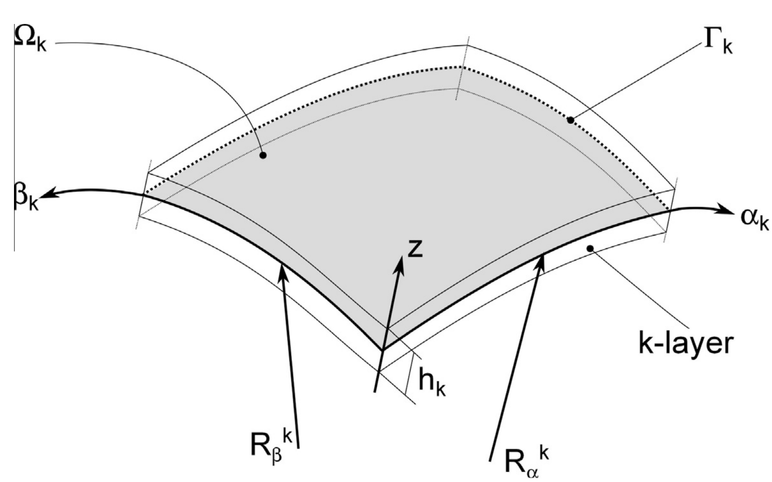

Fig. 1. Shell geometry and reference frame.

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