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Non-linear modes of vibration of thin cylindrical shells in composite laminates with curvilinear fibres

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ABSTRACT

A formulation applicable to free, periodic, geometrically non-linear vibrations of thin shallow shells made of composite layers with curvilinear fibres is presented. The modes of vibration of this type of Variable Stiffness Composite Laminated (VSCL) shallow shells are examined in the non-linear regime. Due to the membrane effects and their coupling with bending, the modes of vibration of VSCL shells are more affected by alterations in the curvilinear fibre paths than what was previously found to occur in plates. Indeed, it is discovered that by changing one of the parameters that defines the fibre path – keeping all other properties of the shells unaltered – the degree of softening can be changed, hardening can become softening, the vibration displacement amplitude at which turning points occur can change and the amplitudes of harmonics vary. A significant deduction, which results from the numerical tests, is that modes of vibration that have mode shapes with more half-waves are less likely to experience softening. A geometric explanation for this behaviour, which does not apply only to VSCL shells, is given.

1. Introduction

Composite material panels can be manufactured, with precision and repeatability, using Automated Fibre Placement (AFP) machines [1,2]. With AFP technology it is possible to dispose fibres along a curvilinear path. By curvilinear fibres in shells, it is meant that the fibres would still be curvilinear if the shells were flat. These manufacturing capabilities allow to implement new composite laminated designs, which have as distinctive feature the fact that the stress–strain constitutive relation is not constant in space. This type of composites belongs to a group of Variable Stiffness Composite Laminates (there are other ways of achieving VSCL, as varying the number of plies or the fibre volume fraction [3], but these are not considered here).

The modes of vibration provide an important insight about the dynamic characteristics of any system; by knowing the non-linear modes of vibration, one identifies where resonances are expected and if the structure experiences softening or hardening. Modes of vibration of VSCL plates were analysed in the linear regime in reference [4], and in the non-linear regime in references [5,6]. This paper addresses the modes of vibration of shells reinforced by curvilinear fibres, in the geometrically non-linear regime. Conservative systems are considered here. Each non-linear mode of vibration is determined by a natural frequency, its harmonics and a mode shape, which depend upon the vibration displacement amplitude [7–11].

An original formulation applicable to thin, laminated composite, cylindrical, shallow shells with curvilinear fibres is implemented. This formulation adopts a *p*-version finite element type strategy [11], followed by modal reduction using the mode shapes of vibration of the linear regime. The validity of the approximation proposed is put to test in numerical comparisons with data published by other authors. However, because this is the first study on non-linear modes of VSCL shells, comparisons are made with data published on other structural elements, as isotropic shells and VSCL plates.

Numerical tests are carried out on VSCL shells, and the effect that the fibre path has on the modes of vibration – chiefly, but not only, on the backbone curves – is investigated. It is verified that using curvilinear fibre paths can strongly affect the modes of vibrations in the geometrically non-linear regime, with consequences as large as turning softening spring effect into hardening, or changing the vibration amplitudes at which turning points occur.

2. Formulation

Open, cylindrical shells as the one represented in Fig. 1 are here analysed; therefore, the first principal radius of curvature is ∞. If

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the second principal radius of curvature is represented by R, the undeformed middle surface of the shell can be defined with good approximation, provided that $R > x$ [12], by

$$w^i(x,y) = -\frac{x^2}{2R} \tag{1}$$

Displacements are defined from this reference surface. We notice – this will be important later on to understand the effect of the fibre orientation on the vibrations - that parallel circles and principal normal sections are located in planes parallel to plane xz. The radius of parallel circles is in this paper represented by R. The terminology used here in relation with geometric properties of shells is the one of reference [13], which can be consulted for more details.

In shallow shells, Cartesian coordinates can be adopted and both Lamé parameters are 1, leading to rather simple kinematic relations [14]. For the shell to be shallow, the raise should be small in comparison with the spans, that is $(\partial w^i(x,y)/\partial x)^2 \ll 1$. We follow references [14–16], according to which a shell can be considered to be shallow if $R \geqslant 2a$, corresponding, in the cases here considered, to $(\partial w^i(a/2,y)/\partial x)^2 \leqslant 0.0625$. Numerical tests in [17] indicate that the natural frequencies of a shell where $R = 2a$ can be accurately computed by a shallow shell formulation.

The formulation here presented is of the equivalent single-layer type [18] and employs Kirchhoff–Love's hypothesis [13,19]. A parentheses is here made, to point out that readers interested in an analysis on the particularities of diverse theories applied to non-linear vibrations of laminated, circular cylindrical, closed shells reinforced with straight fibres, can consult reference [20], where forced vibrations are analysed. This analysis is continued in reference [21], where internal resonances in the same type of shells are investigated, using one of the theories discussed in [20]; forced vibrations are again of interest and it is shown that the effect of internal resonances can be very important. In an earlier work on nonlinear vibrations of isotropic and orthotropic laminated circular cylindrical, closed shells [22], different analytical–numerical models are employed. It is also found that modal interactions may significantly influence the non-linear vibrations. If [20–22] are devoted to closed shells, in reference [23] forced non-linear vibrations of composite laminated, open, shallow shells, with "straight" fibres were investigated. The analysis performed in [23] leads to the conclusion that Kirchhoff–Love's hypothesis generally originates reasonably accurate predictions of periodic non-linear oscillations of thin shells.

The displacement components in the x, y and z directions, respectively represented by $u(x, y, z, t)$, $v(x, y, z, t)$ and $w(x, y, z, t)$, are given by

$$\begin{aligned} u(x,y,z,t) &= u^0(x,y,t) - zw^0_{,x}(x,y,t) \\ v(x,y,z,t) &= v^0(x,y,t) - zw^0_{,y}(x,y,t) \\ w(x,y,z,t) &= w^0(x,y,t) \end{aligned} \tag{2}$$

The unknowns of the vibration problem are the three displacement components – in directions x, y and z – at the middle surface, indicated by superscript 0. These displacement components are written as

$$\begin{Bmatrix} u^0(\xi,\eta,t) \\ v^0(\xi,\eta,t) \\ w^0(\xi,\eta,t) \end{Bmatrix} = \begin{bmatrix} \mathbf{ff}^u(\xi,\eta)^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{ff}^v(\xi,\eta)^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{ff}^w(\xi,\eta)^T \end{bmatrix} \begin{Bmatrix} \mathbf{q}_u(t) \\ \mathbf{q}_v(t) \\ \mathbf{q}_w(t) \end{Bmatrix} \tag{3}$$

Symbols ξ and η represent non-dimensional, local coordinates; $\mathbf{ff}^i(\xi,\eta)$ are vectors of shape functions and $\mathbf{q}_i(t)$ are vectors of generalised displacements (with $i = u, v, w$). This formulation can be categorised as of the p-version finite element type. Contrasting to the h-version of the FEM, the mesh of p-version finite element models is not changed when refining the model; instead, the number of shape functions and generalised coordinates over an element is [26]. The shape functions chosen in this work were already tested in several other applications, including [8–12,27–30]; references [8] or [28], for example, may be consulted for more details on these functions. The set of shape functions is said to be hierarchic, because the finite element space S_{p-1}, spanned by polynomial basis functions with degree up to $p-1$, is embedded in the space S_p, spanned by shape functions up to degree p. The reasons that make p-version formulations interesting are: superior convergence rates in many problems [26]; p-elements are not prone to shear locking [9,26] (even if shear locking is not at all an issue here, because Kirchhoff hypothesis is adopted); when the geometry is simple, as occurs here, a single element is enough, discarding the element assemblage step.

If moderately large displacements are considered, the relation between strains and displacements can be written as [14]:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -z & 0 & 0 \\ 0 & 1 & 0 & 0 & -z & 0 \\ 0 & 0 & 1 & 0 & 0 & -z \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^{\mathbf{m}}_0 + \boldsymbol{\varepsilon}^{\mathbf{i}} + \boldsymbol{\varepsilon}^{\mathbf{nl}} \\ \boldsymbol{\chi}^{\mathbf{b}}_0 \end{Bmatrix} \tag{4}$$

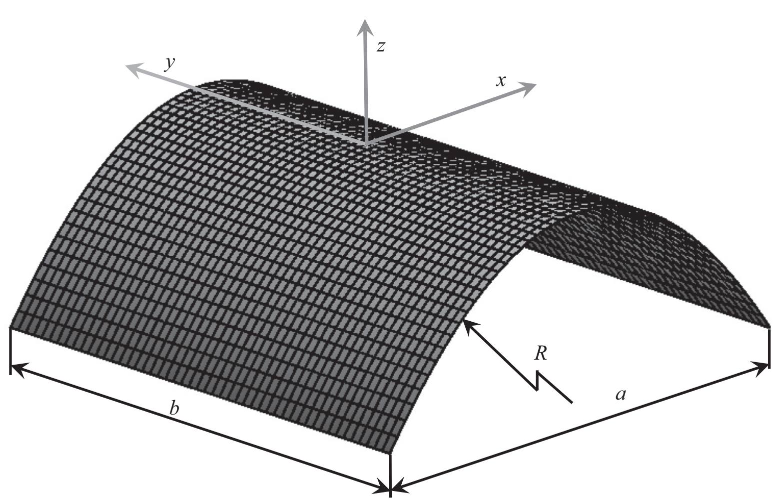

Fig. 1. Representation of an open cylindrical surface, Cartesian reference axis (x, y, z), curvature radius (R), length (a) and width (b) of the projected planform.