Composite Structures 122 (2015) 239–249

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

# Active shape and vibration control for piezoelectric bonded composite structures using various geometric nonlinearities

Shun-Qi Zhang [a,b,*,1], Ya-Xi Li [a,c,1], Rüdiger Schmidt [a]

[a] Institute of General Mechanics, RWTH Aachen University, Templergraben 64, D-52062 Aachen, Germany
[b] School of Mechanical Engineering, Northwestern Polytechnical University, Western Youyi Street 127, 710072 Xi'an, China
[c] School of Aeronautics, Northwestern Polytechnical University, Western Youyi Street 127, 710072 Xi'an, China

ARTICLE INFO

*Article history:*
Available online 27 November 2014

*Keywords:*
Large rotations
Smart structures
Piezoelectric
Shape control
Vibration control

ABSTRACT

This paper deals with simulations of the static and dynamic response, including shape and vibration control, for piezoelectric bonded smart structures using various geometrically nonlinear shell theories based on the first-order shear deformation (FOSD) hypothesis. The nonlinear theories include refined von Kármán nonlinear shell theory (RVK5), moderate rotation shell theory (MRT5), fully geometrically nonlinear shell theory with moderate rotations (LRT5), and fully geometrically nonlinear shell theory with large rotations (LRT56). The structures simulated are mainly comprised of cross-ply or angle-ply laminated thin-walled master structures bonded with isotropic piezoelectric layers or patches that are considered as actuators. Nonlinear finite element (FE) models are constructed for shape and vibration control of structures undergoing large displacements and rotations. Various plates and shells are validated by comparison with those reported in the literature, and then simulated for the current shape and vibration control. A widely used negative proportional velocity feedback control is adopted for the active vibration control. From the simulations, it can be concluded that large rotation theory should be considered for structures undergoing deformations beyond the range of moderate rotations. Additionally, the results show that by applying an appropriate voltage, a desired shape can be achieved, as well as the vibration can be significantly suppressed.



## 1. Introduction

In recent years, piezoelectric sensors and actuators have been increasingly used in active shape and vibration control for thin-walled structures. Bailey et al. [1] performed earlier active vibration control of a cantilever beam with PVDF films. Since then, a large amount of studies on piezoelectric laminated structures for active shape and vibration control were reported in the literature. Most of them modeled smart structures using linear shell theories with 2-dimensional (2D) finite elements based on various through-thickness displacement distribution hypotheses, e.g. Bernoulli beam theory [1,2], Timoshenko beam theory [3], classical plate/shell theory [4,5], first-order shear deformation (FOSD) hypothesis [6], third- or higher-order shear deformation (TOSD or HOSD) hypothesis [7].

However, due to small thickness and weak damping effect, external static or dynamic excitations can cause large deformations of thin-walled smart structures, still in the range of small strains. In such a case, linear theories cannot predict the structural response accurately. Therefore, the consideration of geometric nonlinearities in simulation of active shape and vibration control has been of great interest. Amongst all nonlinear theories, von Kármán type nonlinear theory is the most frequently used one, since it is accurate enough for most structures with moderately large displacements, which can be found e.g. in [8–10] for shape control and in [11,12] for vibration control. Additionally, Kapuria and Alam [13] developed a 1-dimensional (1D) von Kármán zigzag theory for buckling analysis of piezoelectric beams. Following the work of [14,15], Lentzen et al. [16] implemented a moderate rotation nonlinear shell theory for sensing and active vibration control problems. These two nonlinear theories consider only simplified nonlinear strain–displacement relations, yielding that they are applicable only in the range of moderately large displacements and rotations.

In order to include all nonlinearities, Gao and Shen [17] developed earlier a 2D fully geometrically nonlinear FE model for

* Corresponding author at: Institute of General Mechanics, RWTH Aachen University, Templergraben 64, D-52062 Aachen, Germany. Tel.: +49 241 8098286; fax: +49 241 8092231.
*E-mail address:* shunqi@iam.rwth-aachen.de (S.-Q. Zhang).
[1] Both authors contributed equally to this work.

http://dx.doi.org/10.1016/j.compstruct.2014.11.031
0263-8223/

vibration control, and later similar fully geometrically nonlinear theories were implemented into simulation of actuation problems by Kundu et al. [18], Behjat and Khoshravan [19]. In their models, fully geometric nonlinearities were included, but there was no proper rotation updating procedure. This will restrict the application of the theory to only moderate rotations, which predicts not much better structural response than moderate rotation nonlinear shell theory.

To extend the application of fully geometrically nonlinear theory in the range of large displacements and rotations, a finite rotation beam theory was applied by Chróścielewski et al. [20,21], and a fully geometrically nonlinear shell theory with unrestricted finite rotations was implemented by Zhang and Schmidt [22–24] for sensing problems of isotropic layered or composite laminated smart structures. This large rotation nonlinear FE model is equivalent to 3-dimensional (3D) FE models with fully geometric nonlinearities, which can be found in e.g. [25–27].

Following the previous work of Zhang and Schmidt [22–24], this paper investigates simulations of shape and vibration control for piezoelectric integrated smart structures using various nonlinear shell theories. The numerical examples focus on cross-ply or angle-ply laminated structures with bonded isotropic piezoelectric patches or layers. As most researchers did in the literature, the paper considers negative proportional velocity feedback control, but with fully geometric nonlinearities.

## 2. Geometrically nonlinear FE models

### *2.1. Strain field and DOFs*

Using the assumption of an inextensible shell director and the FOSD hypothesis, the Green–Lagrange strain tensor considering fully geometric nonlinearities is given as (see [22,24,28–30])

$$\varepsilon_{\alpha\beta} = \overset{0}{\varepsilon}_{\alpha\beta} + \Theta^3\ \overset{1}{\varepsilon}_{\alpha\beta} + (\Theta^3)^2\ \overset{2}{\varepsilon}_{\alpha\beta}, \tag{1}$$

$$\varepsilon_{\alpha 3} = \overset{0}{\varepsilon}_{\alpha 3}. \tag{2}$$

Here $\varepsilon_{\alpha\beta}$ and $\varepsilon_{\alpha 3}$ ($\alpha, \beta = 1,2$) represent respectively the in-plane and transverse shear strains, and $\Theta^3$ is the distance in the thickness direction. Furthermore, the strain components in (1) and (2) are comprised of linear and nonlinear terms which are listed in Table 1, with using the following abbreviations

$$\overset{n}{\varphi}_{\lambda\alpha} = \overset{n}{v}_{\lambda|\alpha} - b_{\lambda\alpha}\overset{n}{v}_3, \tag{3}$$

$$\overset{n}{\varphi}_{3\alpha} = \overset{n}{v}_{3,\alpha} + b^{\delta}_{\alpha}\overset{n}{v}_{\delta}, \tag{4}$$

$$\overset{n}{v}_{\lambda|\alpha} = \overset{n}{v}_{\lambda,\alpha} - \Gamma^{\delta}_{\lambda\alpha}\overset{n}{v}_{\delta}. \tag{5}$$

All variants are defined in the curvilinear coordinate system ($\Theta^1$, $\Theta^2$ and $\Theta^3$), among which $\overset{0}{v}_i$ ($i = 1,2,3$) are the translational displacements at the mid-surface, and $\overset{1}{v}_i$ are the components of the vector $\overset{1}{\mathbf{u}} = \bar{\mathbf{a}}_3 - \mathbf{n}$, which describes the rotation of the unit normal vector $\mathbf{n}$ of the undeformed mid-surface into the base vector $\bar{\mathbf{a}}_3$ in direction of $\Theta^3$ in the deformed configuration. Furthermore, $b_{\lambda\alpha}$ and $b^{\delta}_{\alpha}$ denote the covariant and mixed components of the curvature tensor, $\Gamma^{\delta}_{\lambda\alpha}$are the Christoffel symbols of the second kind, and $\overset{n}{\square}_{|\alpha}$, $\overset{n}{\square}_{,\alpha}$ ($n = 0,1$) represent the covariant and partial derivative with respect to $\Theta^{\alpha}$.

Including different terms of nonlinearities and different number of parameters results in different nonlinear shell theories, shown in Table 2, which are based on FOSD hypothesis. For more detailed description, we refer to [22,24,30].

Generally, the parameters in all shell theories with the FOSD hypothesis can be expressed by five nodal DOFs, namely, three translational DOFs ($u, v, \omega$) and two rotational DOFs ($\varphi_1, \varphi_2$). If the rotations of the shell director are considerably large, e.g. in LRT56 theory, $\overset{1}{v}_3$ cannot be neglected. Using the two Euler angles, $\varphi_1$ and $\varphi_2$, and the assumption of an inextensible director, the last three parameters, $\overset{1}{v}_i$, are expressed as [22,24,30]

$$\overset{1}{v}_1 = \sin(\varphi_1)\cos(\varphi_2), \tag{6}$$

$$\overset{1}{v}_2 = \sin(\varphi_2), \tag{7}$$

$$\overset{1}{v}_3 = \cos(\varphi_1)\cos(\varphi_2) - 1, \tag{8}$$

where the sixth parameter $\overset{1}{v}_3$ is not negligibly small.

Using the assumption of small or moderate rotations in the simplified nonlinear shell theories, e.g. RVK5, MRT5, LRT5, yields $\sin(\varphi_\alpha) = \varphi_\alpha$ and $\cos(\varphi_\alpha) = 1$. Then these three parameters $\overset{1}{v}_i$ are simplified to

$$\overset{1}{v}_1 = \varphi_1, \quad \overset{1}{v}_2 = \varphi_2, \quad \overset{1}{v}_3 = 0, \tag{9}$$

where the sixth parameter $\overset{1}{v}_3$ disappears. Therefore, these theories have only five parameters.

### *2.2. Finite element model*

#### *2.2.1. Constitutive equations*

In the present paper, linear electro-mechanically coupled constitutive equations are employed, which are given by

$$\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\varepsilon} - \mathbf{e}^{\mathbf{T}}\mathbf{E}, \tag{10}$$

$$\mathbf{D} = \mathbf{e}\boldsymbol{\varepsilon} + \boldsymbol{\epsilon}\mathbf{E}, \tag{11}$$

**Table 1**
Various linear and nonlinear shell theories in FOSD hypothesis.

| Strain | Linear terms | Nonlinear terms |
|---|---|---|
| $2\overset{0}{\varepsilon}_{\alpha\beta}$ | $\underbrace{\overset{0}{\varphi}_{\alpha\beta} + \overset{0}{\varphi}_{\beta\alpha}}_{\text{Linear}}$ | $\underbrace{\overset{0}{v}_{3,\alpha}\overset{0}{v}_{3,\beta}}_{\text{I}}, \underbrace{\overset{0}{\varphi}_{3\alpha}\overset{0}{\varphi}_{3\beta}}_{\text{II}}, \underbrace{\overset{0}{\varphi}{}^{\delta}_{\alpha}\overset{0}{\varphi}_{\delta\beta}}_{\text{III,IV}}$ |
| $2\overset{1}{\varepsilon}_{\alpha\beta}$ | $\underbrace{\overset{1}{\varphi}_{\alpha\beta} - b^{\lambda}_{\beta}\overset{0}{\varphi}_{\lambda\alpha} + \overset{1}{\varphi}_{\beta\alpha} - b^{\delta}_{\alpha}\overset{0}{\varphi}_{\delta\beta}}_{\text{Linear}}$ | $\underbrace{\overset{0}{\varphi}_{3\alpha}\overset{1}{\varphi}_{3\beta} + \overset{1}{\varphi}_{3\alpha}\overset{0}{\varphi}_{3\beta}}_{\text{II}}, \underbrace{\overset{0}{\varphi}{}^{\delta}_{\alpha}\overset{1}{\varphi}_{\delta\beta} + \overset{1}{\varphi}{}^{\delta}_{\alpha}\overset{0}{\varphi}_{\delta\beta}}_{\text{III,IV}}$ |
| $2\overset{2}{\varepsilon}_{\alpha\beta}$ | $\underbrace{-b^{\lambda}_{\beta}\overset{1}{\varphi}_{\lambda\alpha} - b^{\lambda}_{\alpha}\overset{1}{\varphi}_{\delta\beta}}_{\text{Linear}}$ | $\underbrace{\overset{1}{\varphi}_{3\alpha}\overset{1}{\varphi}_{3\beta}}_{\text{II}}, \underbrace{\overset{0}{\varphi}{}^{\delta}_{\alpha}\overset{1}{\varphi}_{\delta\beta}}_{\text{III,IV}}$ |
| $2\overset{0}{\varepsilon}_{\alpha 3}$ | $\underbrace{\overset{1}{v}_{\alpha} + \overset{0}{\varphi}_{3\alpha}}_{\text{Linear}}$ | $\underbrace{\overset{0}{\varphi}{}^{\delta}_{\alpha}\overset{1}{v}_{\delta}}_{\text{II}}, \underbrace{\overset{0}{\varphi}_{3\alpha}\overset{1}{v}_3}_{\text{IV}}$ |

**Table 2**
Strain–displacement relations used by various theories.

| Abbreviation | Description | Terms marked in Table 1 |
|---|---|---|
| LIN5 | Linear shell theory, 5 parameters | Linear |
| RVK5 | Refined von Kármán type nonlinear shell theory, 5 parameters | Linear + I |
| MRT5 | Moderate rotation nonlinear shell theory, 5 parameters | Linear + II |
| LRT5 | Fully geometrically nonlinear shell theory with moderate rotations, 5 parameters | Linear + II + III |
| LRT56 | Fully geometrically nonlinear shell theory with large rotations, 6 parameters | Linear + II + IV |