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Thermal bending of layered composite plates resting on elastic foundations using four-unknown shear and normal deformations theory

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ABSTRACT

A refined plate theory as well as different plate theories is presented to study the thermoelastic response of multilayered cross-ply laminates and angle-ply sandwich plates. The effects of transverse shear strains as well as the transverse normal strain are taken into account. The number of unknown functions involved in the present theory is only four as against six or more in case of other shear and normal deformations theories. The plate is subjected to a sinusoidal temperature distribution and resting on Pasternak's or Winkler's elastic foundation models. The effects due to side-to-thickness ratio, aspect ratio, shear deformation, thermal loads and elastic foundations parameters as well as the variation of lamination angle are investigated.

1. Introduction

The problem of mutual action between two media contains a plate resting on an elastic foundation. Plates supported by elastic foundations are commonly encountered technical problems in many engineering applications. Various kinds of elastic foundation models have been proposed to describe the interaction between the plate structures and their foundations. The simplest one is one-parameter elastic foundation or Winkler's model which regards the foundation as a series of separated spring without coupling effects between each other [1]. This means that there is a proportional interaction between the external forces and the deflection of the applied point in the foundation. This model was improved by Pasternak by adding a shear spring to simulate the interactions between the separated springs in Winkler's model. The two-parameter elastic foundations or Pasternak's model is widely used to describe the mechanical behavior of structure–foundation interactions [2–6].

The bending problems of composite laminates and sandwich plates resting on elastic foundations have received great attention. For this purpose a variety of plate theory has been developed. The classical thin plate theory (CPT) is the simplest one in which the normal to the mid-plane before deformation remains straight and normal to the middle surface after deformation. The CPT is valid only for thin plates on elastic foundations due to ignoring the transverse shear deformation effects [7]. For thick and moderately thick plates, CPT underestimates deflections and stresses of laminates and sandwich plates. To overcome the deficiency of the CPT, many shear deformation plate theories which account for the transverse shear deformation effects have been developed. The first-order shear deformation plate theory (FPT) is presented firstly by Reissner [8] for static plates and Mindlin [9] for dynamics plates. It accounts for the transverse shear effect by the way of linear variation of in-plane displacements through the thickness [6,10,11]. A shear correction factor is required for FPT to compensate for the error due to a constant shear strain assumption through the thickness. So, FSDT is not convenient for use due to the difficulty in determination of the correct value of the shear correction factor. The higher-order shear deformation plate theories (HPT) have been developed to avoid the use of shear correction factor. Reddy [12] has presented the most widely used third-order shear deformation theory due to its high efficiency and simplicity. Other higher-order plate theories such as sinusoidal shear deformation plate theory (SPT) [13–16] and hyperbolic shear deformation plate theory [17] are presented. Additional refined shear deformation plate theories are also presented [18–23].

The advantages of the HPT over the FPT are that the number of independent unknowns is the same as in the FPT, and no shear correction factors are required. In the present article, a refined

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shear and normal deformations plate theory (RPT) is presented. The effects due to transverse shear and normal deformations are both included. Just four independent unknowns are used in the present theory against five independent unknowns used in FPT, HPT and SPT and against six or more independent unknowns used in the corresponding shear and normal deformations theories. The RPT is used to determine accurate solutions for the simply supported cross-ply laminates and sandwich plates resting on elastic foundations. Pasternak's model is used to describe the two-parameter elastic foundation, and getting a special case of Winkler's model by considering a one-parameter elastic foundation. The interaction between the plate and the elastic foundations is considered and included in the equilibrium equations. Numerical results for transverse displacements and stresses are investigated.

2. Geometrical preliminaries

The present study considers a composite rectangular plate of length a, width b and uniform thickness h as shown in Fig. 1. Rectangular Cartesian coordinates (x, y, z) are used and the mid-plane is defined by $z = 0$ and its bounding planes are defined by $z = \pm\frac{1}{2}h(x)$. The plate is composed of n orthotropic layers oriented at angles $\theta_1, \theta_2, \ldots, \theta_n$.

The material of each layer is assumed to possess one plane of elastic symmetry parallel to the x–y plane. Perfect bonding between the orthotropic layers and temperature-independent mechanical and thermal properties are assumed. Let the plate be subjected to a temperature field $T(x, y, z)$.

The in-plane displacements u_1 and u_2, and the transverse displacement u_3 of a material point located at (x, y, z) in the plate are assumed according to the following refined shear deformation plate theory (RPT):

$$\left.\begin{aligned} u_1 &= u(x,y) - z\tfrac{\partial w}{\partial x} + f(z)\tfrac{\partial \varphi}{\partial x}, \\ u_2 &= v(x,y) - z\tfrac{\partial w}{\partial y} + f(z)\tfrac{\partial \varphi}{\partial y}, \\ u_3 &= w(x,y) + g(z)\varphi(x,y). \end{aligned}\right\} \tag{1}$$

The above displacement field contains only four unknown functions u, v, w, and φ. The effects due to transverse shear strain and normal deformations are both included. The function f should be odd function of z while g should be even function of z. The displacement field of the three-unknown classical thin plate theory (CPT) is

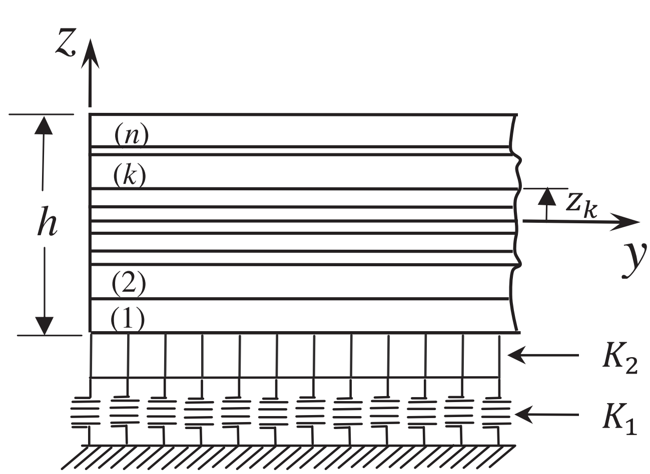

Fig. 1. Schematic diagram for the laminated plate resting on elastic foundations.

obtained easily from Eq. (1) by setting $f(z) = g(z) = 0$. However, the displacement field of the first-order shear deformation plate theory (FPT) is obtained by setting

$$f(z) = z, \quad g(z) = 0, \quad \frac{\partial \varphi}{\partial x} = u_1, \quad \frac{\partial \varphi}{\partial y} = v_1. \tag{2}$$

In addition, the displacement field of the higher-order shear deformation plate theory (HPT) [12] is obtained by setting

$$f(z) = z\left[1 - \frac{4}{3}\left(\frac{z}{h}\right)^2\right], \quad g(z) = 0, \quad \frac{\partial \varphi}{\partial x} = u_1, \quad \frac{\partial \varphi}{\partial y} = v_1. \tag{3}$$

Also, the displacement field of the sinusoidal shear deformation plate theory (SPT) [13–16] is obtained by setting:

$$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right), \quad g(z) = 0, \quad \frac{\partial \varphi}{\partial x} = u_1, \quad \frac{\partial \varphi}{\partial y} = v_1. \tag{4}$$

The number of unknown variables for FPT, HPT and SPT is five. Finally, the present refined four-unknown shear and normal deformations theory (RPT) is given by:

$$f(z) = h\sinh\left(\frac{z}{h}\right) - \frac{4z^3}{3h^2}\cosh\left(\frac{1}{2}\right), \quad g(z) = \frac{1}{4}f'(z). \tag{5}$$

The sinusoidal shear deformation plate theory (SPT) is presented by Touratier [13] for composite plates and it is extended to functionally graded plates by Zenkour [16]. Recently, many investigators have used the same displacement field with the same or different forms of the function $f(z)$ or/and $g(z)$ [18–23]. Note that the present SPT, as well as HPT, is simplified by enforcing traction-free boundary conditions at the surfaces of the plate. The SPT accounts according to a cosine-law distribution of the transverse shear deformation through the thickness of the composite laminated plate. The SPT, HPT, and FPT contain the same number of independent unknowns. No transversal shear correction factors are needed for both SPT and HPT because a correct representation of the transversal shearing strain is given.

In addition, the applied temperature distribution T is assumed by

$$T(x,y,z) = T_0(x,y) + \frac{z}{h}T_1(x,y) + \frac{f(z)}{h}T_2(x,y), \tag{6}$$

where T_i are the transverse temperature loads.

Also, the load–displacement relation between the plate and the supporting foundations is given according to the two-parameter Pasternak's model by

$$R = K_1 w - K_2 \nabla^2 w, \tag{7}$$

where R is the foundation reaction per unit area, K_1 and K_2 are Winkler's and Pasternak's foundation stiffnesses, respectively, and ∇^2 represents Laplace operator. Winkler's model is simply obtained when $K_2 = 0$.

The six strain–displacement components ε_{ij} compatible with the displacement field in Eq. (1) are

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{8}$$

while the six stress–strain relations for a linear elastic plate are given by

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}^k = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{45} & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_1 - \alpha_1\Delta T \\ \varepsilon_2 - \alpha_2\Delta T \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 - \alpha_6\Delta T \end{Bmatrix}^k, \tag{9}$$