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Evaluation of non-linear buckling loads of geometrically imperfect composite cylinders and cones with the Ritz method

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ABSTRACT

A semi-analytical model to predict the non-linear behavior of unstiffened cylinders and cones considering initial geometric imperfections and various loads and boundary conditions is presented. The formulation is developed using the Classical Laminated Plate Theory (CLPT) and Donnell's equations, solving for the complete displacement field. The non-linear static problem is solved using a modified Newton–Raphson algorithm with line-search. A numerical integration scheme for the non-linear matrices is proposed and details regarding the implementation of the proposed method are given. Two methods to include measured imperfections into the analyses are presented and for one method the effect of using different approximation levels for the imperfection field on the non-linear response is investigated, and a minimum approximation accuracy that should be used is determined. The semi-analytical results are verified using finite elements and previous models from the literature. The implemented routines are distributed on-line and are based on a matrix notation simply applicable to other problems.

1. Introduction

Geometric and load imperfections are among the most significant parameters affecting the load carrying capacity of cylinders and cones, as already recognized by the first authors dealing with the topic, e.g. Southwell [1]. Koiter [2] and Donnell and Wan [3] were probably the first authors to accurately take into account the effect of initial geometric imperfections on the non-linear buckling behavior of isotropic cylinders, having developed their contributions independently. Koiter's studies and later Almroth [4] studies were limited to axisymmetric imperfections, usually limited to vanishingly small imperfection amplitudes [5]. The increasing application of composite structures, especially for aerospace and space structures motivated the development of more refined theories applicable to orthotropic materials. In this context, Tennyson [6] presented a thorough review about the first studies developing semi-analytical models for orthotropic materials, all of them constraining the equations for symmetric or anti-symmetric laminates. Simitses et al. [7] are among the first authors investigating the effect of initial imperfections for composite cylinders, followed by Arbocz [8] and Yamada et al. [9]. For conical shells the studies of Goldfeld et al. [10] and Goldfeld [11] are among the most relevant taking into account an initial imperfection field using Koiter's theory with the asymptotic expansion of Budiansky and Hutchinson [12] and assuming a geometric imperfection proportional to the critical buckling mode. Sofiyev and Kuruoglu [13] use a modified Donnell-type of equations and propose an analytical solution for the axial buckling of laminated cones restricted to orthotropic laminates, which results in the limitations already pointed out in Castro et al. [14].

The approaches proposed by Arbocz [8] and Yamada et al. [9], among the references above listed, not limited to symmetric or axisymmetric shaped imperfections. The harmonic function proposed by Yamada et al. [9] to approximate the geometric imperfection field do not include a full Fourier series for the circumferential coordinate, and therefore the function is not capable to represent a general imperfection pattern such as those presented by Degenhardt et al. [15]. Arbocz's approach has the potential to investigate

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the imperfection sensitivity of isotropic and composite shells for various load conditions and using a general imperfection shape, but the studies presented in Ref. [8] are also limited to axisymmetric imperfections and the effects of different boundary conditions are not included.

Castro et al. [16] compared the effect of different imperfection patterns on the non-linear buckling response of cylindrical shells showing that axisymmetric and buckling mode-shaped patterns can be treated as worst-case imperfections leading to the smallest load carrying capacity when compared to measured geometric imperfections. Hence, in order to avoid overly-conservative predictions, it is desirable to take into account more realistic measured imperfection patterns, especially with the growing application of modern measurement systems to obtain such imperfection patterns [17].

In the present study three cylinders presented by Degenhardt et al. [15] and their corresponding measured imperfection patterns are selected and evaluated with the proposed semi-analytical model and with finite elements. The analytical part of this method consists on the integration of the linear stiffness matrices, while the non-linear stiffness matrices and the solution of the non-linear system of equations are performed numerically.

For the semi-analytical model the half-cosine function proposed by Arbocz [18] is applied to approximate the imperfection field, and for the finite element model additionally to the half-cosine function an inverse-weighted interpolation presented in Ref. [16] is used. Due to the limited imperfection data for conical shells the imperfections from the cylinders were mapped to a conical shell in order to verify the accuracy of the proposed method for cones. Different levels of accuracy are used to approximate the imperfections in order to investigate the effect of this measure on the calculated buckling loads. The proposed semi-analytical model uses the Classical Laminated Plate Theory (CLPT) and the Donnell's equations to solve the full displacement field for axial compression, torsion and pressure loads under several classical boundary conditions.

2. Geometric imperfection model

2.1. Measured imperfection patterns

Degenhardt et al. [15] presents a stochastic study where ten replicates of cylinder Z07 presented by Hühne et al. [19] were manufactured and tested. The corresponding geometric imperfections were measured using a photogrammetric system ATOS (automatic transformer observing system) [17], which has a precision of about 0.02 mm, producing an imperfection data that consists on a text file containing the spatial position of the measured points. Among the tested cylinders three are selected for the present study: Z23, Z25 and Z26; with the geometric data and material properties shown in Tables 1 and 2, where R_1 is the radius at the bottom edge, H is the height and α the cone semi-vertex angle. For cylinders Z23, Z25 and Z26 the imperfection data contains 341,009, 340,357 and 331,307 points, respectively, and a representation of the measured imperfection patterns for the cylinders is given in Fig. 1, with the amplitude ξ varying from $\xi_{min} = -0.211$ mm (dark blue) to $\xi_{max} = +0.211$ mm (dark red). A small length between 5 and 10 mm at each side, close to the edges, is not covered by the measurement system such that in all cases the imperfection pattern is stretched to fit the total length of 500 mm.

Table 2
Material properties.

Reference	E_{11} (GPa)	E_{22} (GPa)	ν_{12}	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)
[15]	142.50	8.700	0.28	5.100	5.100	5.100

2.2. Approximating the imperfection field

Arbocz [18] proposed a half-wave cosine function for the imperfection field, which can be written as:

$$w_0 = \sum_{j=0}^{n_0} \sum_{i=0}^{m_0} cos\left(\frac{i\pi x}{L}\right)(A_{ij} cos(j\theta) + B_{ij} sin(j\theta)) \tag{1}$$

where A_{ij} and B_{ij} are the amplitudes of the corresponding functions. The derivatives $w_{0,x}$ and $w_{0,\theta}$ used in the non-linear kinematic equations are:

$$\begin{aligned} w_{0,x} &= \sum_{j=0}^{n_0} \sum_{i=0}^{m_0} -\frac{i\pi}{L} sin\left(\frac{i\pi x}{L}\right)(A_{ij} cos(j\theta) + B_{ij} sin(j\theta)) \\ w_{0,\theta} &= \sum_{j=0}^{n_0} \sum_{i=0}^{m_0} cos\left(\frac{i\pi x}{L}\right) j(-A_{ij} sin(j\theta) + B_{ij} cos(j\theta)) \end{aligned} \tag{2}$$

The coefficients A_{ij} and B_{ij} are calculated based on measured data using for example a linear least-squares algorithm. In theory, m_0 and n_0 can be chosen for any required accuracy, but in practice the least-squares algorithms require a high amount of computer memory, limiting the maximum values for m_0 and n_0. In the present study it is proposed that an optimal relation between m_0 and n_0 can be found such that the approximation functions achieve a higher accuracy for the same required memory, assuming that the measured imperfections have the same pattern along x and θ. For the three measured samples (Z23, Z25 and Z26) the following geometric ratio holds: $2\pi R_1/H \approx 3$, and from Eq. (1) it can be seen that the cosine functions of x have the half the frequency than the trigonometric functions of θ, such that a good approximation is expected with $n_0 \approx (3/2) \times m_0$ and no accuracy gain would be obtained using a higher m_0/n_0 ratio. For a general structure the relation given in Eq. (3) can be used, where R_1 and R_2 are respectively the bottom and top radii:

$$n_0 = \frac{\pi(R_1 + R_2)}{2H} m_0 \tag{3}$$

A second strategy that allows more accurate approximations for w_0 is the use of reduced sample sizes when treating the measured imperfection data. As already mentioned, for cylinders Z23, Z25 and Z26 the imperfection data contains 341,099, 340,357 and 331,307 points, respectively, such that using double precision (64 bits for each entry) will result in a coefficient matrix of $size = m_0 \times n_0 \times 5.4$ MB, which would limit the maximum number of

Table 1
Geometric and laminate data.

Cone/cylinder name	References	R_1 (mm)	H (mm)	α(degrees)	Ply thickness (mm)	Lay-up from inwards to outwards
Z23	[15]	250	500	0	0.1195	(+24/−24/+41/−41)
Z25	[15]	250	500	0	0.1170	(+24/−24/+41/−41)
Z26	[15]	250	500	0	0.1195	(+24/−24/+41/−41)
C02	None	400	200	45	0.125	$(+30/-30/-60/+60/\bar{\mathbf{0}})_S$ [a,b]

[a] Middle plies marked with a bar, e.g.: $\bar{\mathbf{0}}$.
[b] The subscript S indicates a symmetric lay-up.