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Optimum tailoring of fibre-steered longitudinally stiffened cylinders

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ABSTRACT

A computationally efficient framework is presented to design fibre steered laminates in cylindrical shells, stiffened with longitudinal stiffeners, optimally. The framework is comprised of a semi-analytical analysis approach and a multi-step optimisation framework, and is applied for maximum buckling capacity design of fibre steered laminates of two stiffened cylinders, with different thickness to radius ratios, under bending moment. If the maximum buckling capacity designs are material failure critical, strength is imposed as a constraint in the optimisation problem. Comparisons of buckling capacity of steered fibre laminates with those of straight fibre laminates are shown and validated. Finally, to gain in-depth understanding of the way buckling capacity is improved by fibre steering, the axial section load and the critical buckling modes of straight and steered fibre laminates are compared.

1. Introduction

Variable stiffness, VS, design of composite laminates through fibre steering using automated fibre placement machines has further extended the traditional design boundaries allowing to use the directional properties of composites optimally [1]. VS laminate design requires advanced modelling, analysis and optimisation techniques, some of which are reviewed in reference [2]. In practice, thin laminated structures are usually reinforced with stiffeners, however, the amount of research on VS laminated stiffened structures is limited. In one of the rare studies, Coburn et al. [3] develop an analytical method for the buckling analysis of a novel blade stiffened VS laminate which is suitable for future research on the design and optimisation of such laminates.

Steered fibre cylindrical shells have been the subject of several studies [4–8]. The authors have developed a computationally efficient framework to optimally design steered fibre laminated cylindrical shells with general cross-sectional geometry [9]. The framework includes a semi-analytical finite difference (SAFD) technique for static and buckling analysis of unstiffened cylindrical shells with arbitrary cross-sections. Compared to full finite element analysis, the SAFD technique reduces the analysis time through a reduction of number of degrees of freedom (DOFs); and is thus suitable for optimisation purposes. In this paper, the SAFD technique is extended to longitudinally stiffened cylindrical shells and the design framework is implemented to design two stiffened cylinders with steered fibre laminates. The obtained designs are investigated to achieve a better understanding of the interaction between the stiffeners and fibre steering patterns and reveal the mechanisms through which fibre steering improves structural performance in the presence of stiffeners.

2. Analysis

The static and buckling problems of longitudinally stiffened cylindrical shells are formulated variationally from the total potential energy of the cylindrical shell and the stiffeners. The total potential energy has to be expressed in terms of the DOFs of the unstiffened cylindrical shell, therefore the stiffener-shell kinematic relations must be determined.

2.1. Stiffener-shell kinematic relations

A typical cross-section of a stiffened cylindrical shell is depicted in Fig. 1. Assuming that the stiffeners are perfectly bonded to the cylindrical shell and their cross-section is rigid, the translational, u^s, v^s, w^s, and rotational, $\theta^s_\beta, \theta^s_\alpha, \theta^s_n$, DOFs of any point on the stiffener cross-section are related to the corresponding, $u, v, w, \theta_\beta, \theta_\alpha, \theta_n$, DOFs of the reference point on the middle surface of the cylindrical shell through the following equations:

$$\begin{bmatrix} \theta^s_\beta \\ \theta^s_\alpha \\ \theta^s_n \end{bmatrix} = \begin{bmatrix} \theta_\beta \\ \theta_\alpha \\ \theta_n \end{bmatrix}, \quad \begin{bmatrix} u^s \\ v^s \\ w^s \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} \theta_\beta \\ \theta_\alpha \\ \theta_n \end{bmatrix} \times \begin{bmatrix} 0 \\ \beta^s \\ z^s \end{bmatrix} \tag{1}$$

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where β^s and z^s are the distances of the point on the stiffener cross-section from the reference point on the middle surface of the cylindrical shell in the $\mathbf{i}_\beta$ and $\mathbf{i}_n$ directions. The drilling DOF is not defined for the shell and hence $\theta_n^s = \theta_n = 0$.

The state of strain in the longitudinal stiffeners under extension, bending and torsion is constant along the beam and can be expressed in terms of the strain state of the reference point on the middle surface of the cylindrical shell by using Eq. (1) and Sanders' strain–displacement relations [9]:

$$\epsilon_\alpha^s = \frac{\partial u^s}{\partial \alpha} = \epsilon_\alpha + z^s\kappa_\alpha, \quad \kappa_\alpha^s = \frac{\partial \theta_\alpha^s}{\partial \alpha} = \kappa_\alpha, \quad \kappa_n^s = \frac{\partial \theta_n^s}{\partial \alpha} = 0,$$
$$\tau^s = \frac{\partial \theta_\beta^s}{\partial \alpha} = \frac{1}{2}\tau + \frac{1}{4R}\left(\epsilon_\alpha + \frac{\partial v}{\partial \alpha}\right) \tag{2}$$

where $\epsilon_\alpha, \kappa_\alpha, \kappa_n$ and τ are the axial strain, changes of curvature around the $\mathbf{i}_\beta$ and $\mathbf{i}_n$ directions, and twist, respectively, and R is the radius of curvature of the cylinder in the $\mathbf{i}_\beta$ direction at the reference point.

2.2. Static analysis

The strain energy of a stiffener is formulated as:

$$\mathcal{U}^s = \frac{L}{2}\int_{A^s}\left(\begin{bmatrix}\epsilon_n^s & \epsilon_s^s\end{bmatrix}\begin{bmatrix}E & \epsilon_n^s \\ G & \epsilon_s^s\end{bmatrix}\right)dA \tag{3}$$

where L and A^s are the stiffener length and cross-sectional area, E and G are the normal and shear elastic moduli. In Eq. (3), ϵ_n^s is the strain in the normal direction to the cross-section induced by extension or compression and bending and ϵ_s^s is the shear strain induced by torsion, defined as:

$$\begin{bmatrix}\epsilon_n^s \\ \epsilon_s^s\end{bmatrix} = \begin{bmatrix}1 & y_l & 0 \\ 0 & 0 & r_l\end{bmatrix}\begin{bmatrix}\epsilon_\alpha^s \\ \kappa_\alpha^s \\ \tau^s\end{bmatrix} \tag{4}$$

where r_l is the distance between the centroid of the stiffener cross-section and the selected point on the stiffener cross-section and y_l is the component of r_l in the $\mathbf{i}_n$ direction of the local coordinate system in Fig. 1. For composite laminated stiffeners $E = 1/(A^{*-1})_{11}$ and $G = 1/(D^{*-1})_{66}$ where $\mathbf{A}^* = (1/h)\mathbf{A}, \mathbf{D}^* = (12/h^3)\mathbf{D}$ and h is the laminate thickness. Using Eq. (2) and Sanders' strain–displacement relations [9], the stiffener strain state is expressed in terms of the DOFs of the shell at point $j, \mathbf{U}_j$:

$$\begin{bmatrix}\epsilon_\alpha^s \\ \kappa_\alpha^s \\ \tau^s\end{bmatrix} = \mathbf{B}_j^s\mathbf{U}_j \tag{5}$$

where $\mathbf{B}_j^s$ and $\mathbf{U}_j$ are defined in Appendix A.

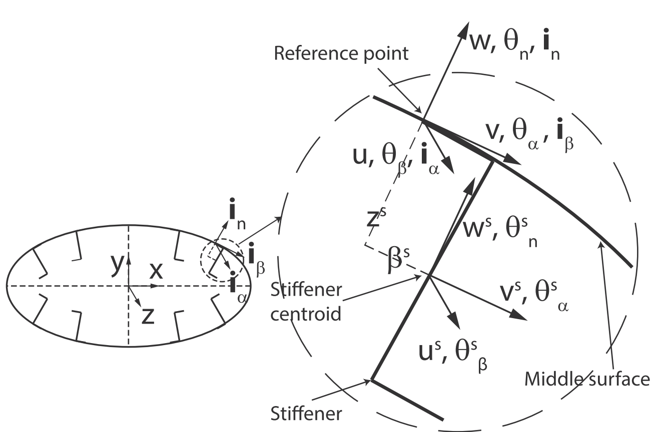

Fig. 1. The cross-section of a stiffened cylinder, DOFs of the reference point on the shell middle surface and DOFs of the stiffener centroid.

Using Eqs. (4) and (5) in Eq. (3), the potential energy of a stiffener attached to the jth discretisation point on the shell is expressed as:

$$\mathcal{U}^s = \frac{L}{2}(\mathbf{U}_j)^T(\mathbf{B}_j^s)^T\begin{bmatrix}EA^s & EQ_x^s & 0 \\ EQ_x^s & EI_x^s & 0 \\ 0 & 0 & GJ^s\end{bmatrix}\mathbf{B}_j^s\mathbf{U}_j \tag{6}$$

where Q_x^s and Q_y^s are the first moments of area of the stiffener cross-section about $\mathbf{i}_\beta$ and $\mathbf{i}_n$, respectively, which are equal to zero since the local coordinate system is placed on the centroid of the stiffener cross-section, I_x^s, I_y^s and I_{xy}^s are the second moments of area and J^s is the polar moment of area of the stiffener cross-section. The stiffness matrix from the contribution of this stiffener is:

$$\mathbf{k}^s = \frac{L}{2}(\mathbf{B}_j^s)^T\begin{bmatrix}EA^s & EQ_x^s & 0 \\ EQ_x^s & EI_x^s & 0 \\ 0 & 0 & GJ^s\end{bmatrix}\mathbf{B}_j^s \tag{7}$$

As described in Appendix A, the $\mathbf{k}^s$ matrices from all the stiffeners are assembled into the stiffness matrix of the cylindrical shell [9], $\mathbf{K}$, to build the stiffness matrix of the stiffened shell.

2.3. Buckling analysis

The eigenvalue buckling problem is formulated by assuming moderately large rotations for the shell mid-surface and adding the von Karman nonlinear term to the normal strain. Therefore, the stiffener strain energy is:

$$\mathcal{U}^s = \frac{L}{2}\int_{A^s}\begin{bmatrix}\epsilon_n^s + \frac{1}{2}(\theta_\alpha^s)^2 & \epsilon_s^s\end{bmatrix}\begin{bmatrix}E\left(\epsilon_n^s + \frac{1}{2}(\theta_\alpha^s)^2\right) \\ G\epsilon_s^s\end{bmatrix}dA \tag{8}$$

Using Eq. (2), Sanders' strain–displacement relations [9] and assuming sinusoidal variation in the axial direction, m [9], the stiffener strain state is expressed in terms of buckling DOFs of the shell at point $j, \mathbf{a}_j$:

$$\begin{bmatrix}\epsilon_\alpha^s \\ \kappa_\alpha^s \\ \tau^s\end{bmatrix} = \mathbf{G}_j^s\mathbf{a}_j e^{(im\pi\alpha/L)}, \quad \theta_\alpha^s = \boldsymbol{\Gamma}_j^s\mathbf{a}_j e^{(im\pi\alpha/L)} \tag{9}$$

where $\mathbf{G}_j^s, \boldsymbol{\Gamma}_j^s$ and $\mathbf{a}_j$ are defined in Appendix B, i is the imaginary unit and α is the axial surface parameter [9].

Using Eqs. (4) and (9) in Eq. (8), the potential energy of a stiffener attached to the jth discretisation point on the cylinder cross-section is:

$$\mathcal{U}^s = \frac{L}{2}(\mathbf{a}_j)^T\left[(\mathbf{G}_j^s)^T\begin{bmatrix}EA^s & EQ_x^s & 0 \\ EQ_x^s & EI_x^s & 0 \\ 0 & 0 & GJ^s\end{bmatrix}\mathbf{G}_j^s + (\boldsymbol{\Gamma}_j^s)^T N_n^s\,\boldsymbol{\Gamma}_j^s\right]\mathbf{a}_j \tag{10}$$

Taking the second derivative of the strain energy of the stiffener gives rise to the following material and geometric stiffness matrices:

$$\mathbf{k}^{ms} = \frac{L}{2}(\mathbf{G}_j^s)^T\begin{bmatrix}EA^s & EQ_x^s & 0 \\ EQ_x^s & EI_x^s & 0 \\ 0 & 0 & GJ^s\end{bmatrix}\mathbf{G}_j^s, \quad \mathbf{k}^{gs} = \frac{L}{2}(\boldsymbol{\Gamma}_j^s)^T \quad N_n^s \quad \boldsymbol{\Gamma}_j^s \tag{11}$$

These matrices will be assembled at the discretisation point j to the global material, $\mathbf{K}^m$, and geometric, $\mathbf{K}^g$, stiffness matrices as explained in Appendix B.

3. Multi-step optimisation framework

Based on initial work by IJsselmuiden et al. [10] to design a multi-patch laminated panel, a multi-step optimisation framework