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Upper and lower bound buckling load of perfect and delaminated fiber-reinforced composite columns

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ABSTRACT

In this paper, a re-look into the upper and lower bound solution to the buckling of composite columns is presented in details and this includes the complete derivation of the expressions for the kinematical compatibility conditions. Included in this contribution are also the results of the current finite element simulation together with the results of the experiments. Moreover, further investigations are conducted to test whether the proposed three methods of calculating the effective flexural stiffness can be employed in cooperation with the developed mathematical model to form the desired upper and lower bounds for the buckling loads of both the perfect and single-delaminated fiber-reinforced composite beams for the various parameters that are studied. In these analyses, single-delaminated fiber-reinforced composite beams which consist of random ply-orientations and stacking sequences are employed to put up a stringent test on the generalness and applicability of these three methods. It is observed that for all the cases that were examined, these three means of evaluating the effective bending modulus of a fiber-reinforced composite beam pass are validated.

1. Introduction

The fiber-reinforced composite materials are used widely because of their high strength-to-density and stiffness-to-density ratios as compared with most metals. This is especially true in aerospace industries such as space applications, the making of military and civil aircrafts, where weight saving is very important. Other areas include automotive and sports applications. Furthermore, the cost of fiber-reinforced composite materials has decreased over the years. This is due to the increased manufacturing experience accumulated over the years and more effective manufacturing technologies for mass production. However, fiber-reinforced composites are well known to be susceptible to delamination. Once delamination occurs in the composite laminate, stresses within the composite laminate will be redistributed. This will then lower the compressive load carrying capacity of the composite laminate. The causes for occurrence of delamination include the high inter-laminar stresses at the free edge, impact and fabrication defects.

A lot of researchers have investigated on the effects of delamination on the critical load of both isotropic and fiber-reinforced composite laminates. Simitses [1] and Simitses et al. [2] created a one-dimensional model to simulate a laminate plate with a through width delamination. This model was believed to be sufficient to describe the behavior of the laminate plate. Simitses [1] also considered delamination growth while Simitses et al. [2] utilized the thin-film analysis for comparison. Kapania and Wolfe [3] examined the buckling behavior of a beam-plate with two delaminations of equal length, one above the other. The delaminations were also taken to be through-the-width. Lee et al. [4] employed the layer-wise theory to examine the buckling behavior of a beam-plate with multiple delaminations under compression. Lim and Parsons [5] developed a numerical model in their paper to calculate the buckling load of a composite beam with multiple delaminations quickly, in which both the Lagrange multipliers and Rayleigh–Ritz energy method were employed. Shu [6] utilized the classical beam theory to model a beam with double delaminations into five interconnected segments. Furthermore, two coordinate systems were used and the gradient was utilized as the unknown, which greatly decreased the complexity of the solution. Wang et al. [7] used a continuous analysis to examine the buckling loads of delaminated beams and plates. The Stoke's transformation technique was used to investigate the buckling behavior of clamped and simply supported orthotropic beam-plates. Gaudenzi [8] employed a classical nonlinear finite element method utilizing

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the linearized buckling analysis to examine the delamination buckling in practical applications. Huang and Kardomateas [9] employed suitable kinematical continuity conditions, equilibrium equations and boundary conditions with the linearized nonlinear differential equation to derive a closed form solution for a plate with two delaminations. Kim and Hong [10] studied both cases of composite laminate with either a through-the-width or an embedded delamination. In addition, both buckling and post-buckling were examined. A geometrically nonlinear finite element analysis was used in the post-buckling state of the laminate to deal with the geometric complexity and large deformation. Lifshitz and Gildin [11] conducted experiments to investigate how the carbon/epoxy composite beam behaved under cyclic compressive loading. Firstly, the effect of a delamination on the fatigue life was studied. Then, the respond of the composite beam from the initial cyclic compressive loading to failure was examined. Kyoung et al. [12] examined the buckling and post-buckling behaviors of laminates with multiple through-the-width delaminations in their paper. Furthermore, these delaminations were modeled in a more realistic configuration. Kyoung et al. [13] studied both one-dimensional composite laminates with multiple through-the-width delaminations and two-dimensional cross-ply composite laminates with circular delaminations. Jones [14] published an excellent book on both the micromechanics and macromechanics aspects of fiber-reinforced composite materials and structural phenomena. Naik and Ramasimha [15] explored the buckling behavior of the typical woven fabric composites with a central delamination under axial compression. The classical lamination theory, equilibrium of forces and finite element analysis were involved in their approach. Parlapalli and Shu [16] examined the buckling behavior of a beam that was made of two different materials with an asymmetry delamination. New non-dimensionalized parameters, axial and bending stiffnesses and effective-slenderness ratio were introduced. Yap and Chai [17] employed two methods of computing the effective flexural stiffness into a mathematical model to predict the buckling load of fiber-reinforced composite beam with a single delamination. Both linear and nonlinear finite element simulations were performed. Yap and Chai [18] derived a closed form expression to calculate the effective flexural modulus of a fiber-reinforced composite beam. Then, together with the two ways of determining the effective flexural stiffness used by Yap and Chai [17] in their earlier work, they managed to come up with the upper and lower bounds for the buckling loads of both the perfect and single-delaminated composite beams. Chai and Yap [19] applied the same effective flexural stiffness closed formed solution to investigate the effect of various coupling terms on the bending, buckling and free vibration of perfect fiber-reinforced composite laminated beam. Chai et al. [20] further looked into the structural responses of a perfect fiber-reinforced composite beam under both axial force and combined axial plus transverse loads for the first time.

In view of queries for the use of incremental forces in the analytical model, an attempt has been made here to explain the utilization of these incremental forces in the formulation of the mathematical model in detail with illustrations. Moreover, for both Refs. [17,18], the finite element simulation results were used to verify their different methods of computing the effective flexural modulus of a fiber-reinforced composite beam without proper substantiation. Hence, in this paper, both the experiments and relevant results from other authors are included to validate their way of performing the finite element simulations. Thirdly, all the cases from Refs. [17–20] only consisted of specially orthotropic, with at most anti-symmetric fiber-reinforced composite beams. Therefore, this paper tries to look into fiber-reinforced composite beams with more general ply-orientations and stacking sequences. This is important because for specially orthotropic and anti-symmetric scenarios, various coupling terms will become zero. This will introduce ambiguity in the usage of the effective flexural stiffness expressions to form the desired upper and lower bounds proposed by Yap and Chai [18]. Moreover, as the closed form formula to calculate the effective flexural modulus of a fiber-reinforced composite beam developed by Yap and Chai [18] and further examined by Refs. [19,20] caters for all the coupling terms in the classical lamination theory, one has to consider the proposed more universal cases in order to put a more stringent test on the applicability of the derived closed form equation to prove its generalness.

2. Mathematics

In this section, the classical lamination theory, the three formulae employed by Refs. [17–20] to compute the effective flexural stiffness of a fiber-reinforced composite beam and the analytical model that can be utilized to work out the buckling load of a single-delaminated fiber-reinforced composite beam will be addressed.

2.1. Classical lamination theory

Jones [14] reveals that in the framework of classical lamination theory, the force and moment resultants can be worked out to be:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^o_{xx} \\ \varepsilon^o_{yy} \\ \gamma^o_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^o_{xx} \\ \varepsilon^o_{yy} \\ \gamma^o_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{2}$$

The membrane strains are defined as

$$\begin{Bmatrix} \varepsilon^o_{xx} \\ \varepsilon^o_{yy} \\ \gamma^o_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_o}{\partial x} \\ \frac{\partial v_o}{\partial y} \\ \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \end{Bmatrix} \tag{3}$$

and curvatures as

$$\begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = -\begin{Bmatrix} \frac{\partial^2 w_o}{\partial x^2} \\ \frac{\partial^2 w_o}{\partial y^2} \\ 2\frac{\partial^2 w_o}{\partial x \partial y} \end{Bmatrix} \tag{4}$$

The A_{ij} are extensional stiffnesses, B_{ij} are bending extension coupling stiffnesses and D_{ij} are bending stiffnesses. They are evaluated as:

$$A_{ij} = \sum_{k=1}^{N} (\overline{Q_{ij}})_k (z_k - z_{k-1}) \tag{5}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} (\overline{Q_{ij}})_k (z_k^2 - z_{k-1}^2) \tag{6}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} (\overline{Q_{ij}})_k (z_k^3 - z_{k-1}^3) \tag{7}$$

Fig. 1 illustrates how the z_{k-1} and z_k are defined. The $(\overline{Q_{ij}})_k$ in equations (5)–(7) are the transformed reduced stiffnesses of the k_{th} layer.