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Reissner's mixed variational theorem-based nonlocal Timoshenko beam theory for a single-walled carbon nanotube embedded in an elastic medium and with various boundary conditions

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A B S T R A C T

The nonlocal Timoshenko beam theories (TBTs), based on the Reissner mixed variational theorem (RMVT) and principle of virtual displacement (PVD), are derived for the analysis of a single-walled carbon nanotube (SWCNT) embedded in an elastic medium and with various boundary conditions, in which the Eringen nonlocal elasticity theory is used. The strong formulations of RMVT- and PVD-based nonlocal TBTs and their associated possible boundary conditions are presented. The interaction between the SWCNT and its surrounding elastic medium is simulated using the Pasternak foundation model. The generalized displacement and force resultant components induced in the loaded SWCNT are obtained using the meshless collocation method, in which the shape functions of the primary variables are constructed using the differential reproducing kernel (DRK) method. The results show that RMVT-based nonlocal TBT is superior to its PVD-based counterpart in that the convergent rate of the RMVT-based nonlocal TBT is faster than that of the PVD-based one, that the predictions of generalized force resultants obtained using the RMVT-based nonlocal TBT are more accurate than those obtained using the PVD-based nonlocal one, and that the highest order of the base functions used in the RMVT-based nonlocal TBT is lower than that used in the PVD-based one.

1. Introduction

Due to their superior stiffness, strength and conductivity properties, carbon nanotubes (CNTs) have been widely used for various nanotechnology applications since their discovery in 1991 [1], such as nanoprobes, nanoactuators, gene delivery systems and reinforcements in polymer composites [2–4]. Comprehensive studies of the structural behaviors of CNTs thus become an important research subject to enhance the performance and lifetime of the related composite devices.

Atomistic modeling is both time consuming and expansive, and nonlocal continuum mechanics thus provides an attractive alternative approach to investigate the static behaviors and dynamic responses of nano-scaled structures, such as CNTs. According to Eringen nonlocal elasticity theory [5–8], nonlocal continuum mechanics mainly differs from local continuum mechanics in that the stresses at a typical material point induced in a loaded elastic body will depend on the strains at only that material point in the latter, while these will depend on all material points in the elastic body in the former due to the small length scale effect. This means that at the nano-scale the lattice spacing between individual atoms becomes increasingly important, and the discrete structure of the material can no longer be homogenized into a continuum. A variety of nonlocal structural models, based on the Eringen nonlocal constitutive relations, have been developed for the static and dynamic analyses of CNTs with and without being embedded in a polymer matrix, and reviewed in the open literature [9–12], such as the nonlocal Euler–Bernoulli beam theory (EBT), nonlocal Timoshenko beam theory (TBT), nonlocal Levison beam theory (LBT), Reddy's [13] and Thai's [14] nonlocal refined beam theories (RBTs) and nonlocal sinusoidal beam theory (SBT) [15], with the nonlocal TBT being more widely used than the others, in which Thai's nonlocal RBT differs from Reddy's nonlocal RBT in that the total normal displacement component is separated into the shear and bending parts. The literature survey carried out in this article will thus focus on the nonlocal TBT-based static and dynamic analyses of CNTs.

Wang et al. [16,17] investigated the bending and buckling of a single- or doubly-walled carbon nanotubes (SWCNTs or DWCNTs) with various boundary conditions using the TBT combined with the Eringen nonlocal constitutive relations for an elastic body, in which the shear deformation and small-scale effects were

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considered. The Euler–Lagrange equations and the possible boundary conditions were derived using the principle of virtual displacement (PVD). The free vibration responses of an SWCNT with different boundary conditions were studied by Ansari and Sahmani [18] using various PVD-based nonlocal classical beam theories in conjunction with the differential quadrature (DQ) method and a molecular dynamics (MD) simulation. Through the fitting of the numerical results of the DQ and MD methods, the appropriate values of the nonlocal parameter (i.e., the length scale parameter) for each nonlocal beam model with different types of chirality and boundary conditions were recommended. Simsek and Yurtcu [19] examined the static and buckling behaviors of a simply-supported, functionally graded (FG) nanobeam on the basis of the nonlocal TBT and EBT. Based on the nonlocal TBT, Pradhan [20] and Roque et al. [21] presented the analyses of nanobeams using the finite element method (FEM) and the meshless method, respectively. In the FEM, a weak formulation related to the nonlocal TBT was derived using the weighted-residual approach, and then was applied to the corresponding analyses of nano-scaled structures, while in the meshless method the strong formulation of the TBT was applied to a series of randomly-scattered sampling nodes and the shape functions of the primary variables were constructed using the global and local radial basis functions [22–24]. The results obtained using the above-mentioned numerical methods were found to be in excellent agreement with the exact solutions available in the literature.

Eltaher et al. [25] and Ghannadpour et al. [26] studied the size-dependent static, free vibration and buckling problems of FG nanobeams using the EBT combined with the FEM and Ritz methods, respectively, in which the equilibrium equations were derived using the PVD, and the material properties of the nanobeam were assumed to vary through the thickness according to the power law. Based on the nonlocal TBT, Ke et al. [27] investigated the nonlinear vibration of piezoelectric nanobeams subjected to an applied voltage and a uniform temperature change, in which a detailed parametric study was undertaken to examine the influences of the nonlocal parameter, temperature change, and external electric voltage on the size-dependent nonlinear vibration characteristics of the piezoelectric nanobeam.

In some nanotechnology applications CNTs are embedded in a polymer matrix to produce CNT-reinforced polymer composite structures. The interaction effect between the CNT and its surrounding elastic medium thus has to be therefore considered in the assorted mechanical analyses of the CNT. Murmu and Pradhan [28–30] presented the thermo-mechanical vibration and thermo-mechanical buckling analyses of an SWCNT embedded in an elastic medium based on the nonlocal EBT and TBT in conjunction with the DQ method, in which the Winkler-type and Pasternak-type foundation models were used to simulate the interaction effect between the CNT and its surrounding elastic medium, and the small scale parameter and the stiffness of the surrounding medium were demonstrated to significantly affect the critical load parameters and natural frequencies of the CNT. Based on a higher-order shear deformation shell theory with von Karman kinematic nonlinearity, Shen and Zhang [31,32] presented the buckling and postbuckling analyses of axially compressed double-walled CNTs, and the results showed that the buckling and postbuckling behaviors of CNTs are very sensitive to the small scale parameter.

After a close literature survey, we found that although the nonlocal continuum model is a compromise it remains an effective analytical tool due to its lower cost and time requirements with regard to the analysis of CNTs with and without being embedded in an elastic medium. Most of the above-mentioned nonlocal continuum models are based on the PVD, rather than the Reissner mixed variational theorem (RMVT) [33,34], even though RMVT-based theories have been demonstrated to be superior to the PVD-based ones for the analysis of macro- and micro-scaled structures with regard to the accuracy and convergence rate by Carrera [35–37], Carrera and Ciuffreda [38,39] and Wu and Li [40,41], Wu and Chiu [42] and Wu et al. [43]. In the PVD, the generalized displacements are regarded as the primary variables subject to variation, while in the RMVT, these are both the generalized displacements and the generalized force resultants. According to Eringen nonlocal elasticity theory, the stress–strain (or the generalized force resultant-displacement) relations of an elastic body are expressed in a system of differential equations, rather than a system of algebraic equations for the local elasticity theory, so that the natural boundary conditions cannot be directly imposed in the analysis when the nonlocal PVD-based continuum models are used. In addition, the determination of the generalized force resultants involves a numerical differentiation process related to the determined primary variables, which is time-consuming and always leads to significant errors.

To the best of the authors' knowledge, the nonlocal beam theories for embedded CNTs published in the open literature are almost based on the PVD, rather than the RMVT. On the basis of the above-mentioned advantages of RMVT-based theories, in this article an RMVT-based nonlocal TBT combined with a meshless collocation method using the differential reproducing kernel (DRK) interpolation [44] is first proposed and developed for the bending analysis of an SWCNT embedded in an elastic medium and with various boundary conditions. The interaction between the SWCNT and its surrounding elastic medium is considered using a Pasternak-type foundation. The Euler–Lagrange equations of RMVT- and PVD-based nonlocal TBT resting on a Pasternak foundation and their associated boundary conditions are derived using the calculus of variation. The optimal values of the parameters selected in the implementations of the meshless DRK interpolation method are examined, such as the highest order of the base functions, and the radius of the influence zones. A comparative study for the generalized displacement and force resultant components induced in the loaded SWCNT obtained using RMVT- and PVD-based is undertaken. Moreover, a parametric study related to the influences of some crucial effects on the static behaviors of the loaded SWCNT is conducted, such as the small scale effect, aspect ratios, different boundary conditions, and the stiffness of the foundation.

2. The RMVT-based local TBT

2.1. Kinematic and kinetic assumptions

For a moderately thick beam with a length L, in the TBT the shear deformation effect is considered to be a constant through the thickness coordinate of the beam, and the related displacement field is given as follows:

$$u_1(x,z) = u(x) - z\phi(x), \quad (1)$$

$$u_2(x,z) = 0, \quad (2)$$

$$u_3(x,z) = w(x) \quad (3)$$

in which $u_i(x,z)$ ($i = 1 - 3$) denote the displacement components of the beam in the x, y and z directions, respectively. $u(x)$ and $w(x)$ stand for the mid-plane displacement components in the x and z directions, and $\phi(x)$ is the total rotation in the $x - z$ plane. The deformation of a section in the $y - z$ plane is shown in Fig. 1(a).

The in-plane and out-of-plane motions of the beam will be uncoupled when the material properties are symmetric with respect to the middle plane, and thus the mid-plane displacement component ($u(x)$) is discarded in the following derivation.

The strain–displacement relations of the beam are given by