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Optimization method for the determination of material parameters in damaged composite structures

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An optimization method to identify the material parameters of composite structures using an inverse method is proposed. This methodology compares experimental results with their numerical reproduction using the finite element method in order to obtain an estimation of the error between the results. This error estimation is then used by an evolutionary optimizer to determine, in an iterative process, the value of the material parameters which result in the best numerical fit. The novelty of the method is in the coupling between the simple genetic algorithm and the mixing theory used to numerically reproduce the composite behavior. The methodology proposed has been validated through a simple example which illustrates the exploitability of the method in relation to the modeling of damaged composite structures.

1. Introduction

The structural use of composite materials is widespread in many fields, including civil infrastructures and the aerospace, automotive and marine industries [1]. The high strength-to-weight and stiffness-to-weight ratios of these materials, in addition to their corrosion resistance and thermal stability, make them well suited for structural applications in which weight reduction is a priority.

Composite materials are made of two or more simple materials or components, typically exhibiting the best qualities of these components and, often, superior properties to those of the individual components alone [2]. In general, composites are designed to meet certain structural needs. The determination of the overall behavior of the composite material is key to the design process. Representing the composite as a single orthotropic material with the averaged properties of the whole set has proven unsatisfactory. The main drawback of this approach is that it cannot capture correctly the behavior of the composite if one or more of its components exceeds the elastic limits [3]. Hence, composite materials need to be modeled using theories that allow taking into account the behavior of the simple materials, which can be quite diverse and include anisotropy, plasticity and damage, among other characteristics. One of the most commonly used is the mixing theory [4], whose general theoretical framework was initially developed by Truesdell and Toupin [5].

The classical mixing theory explains the behavior of a composite material according to the interaction between the components of the composite. It is based on the hypotheses that all components suffer the same strains and each component contributes to the behavior of the composite in the same proportion as their volumetric participation. Each component is a material in itself whose individual behavior can be represented by its own constitutive law. Thus, the mixing theory can be considered a constitutive equation manager. This behavior combination technique allows preserving the original constitutive law of each component, which is especially useful when studying composite structures with the finite element method (FEM).

Finite element analysis has proven to be an extremely useful tool in the design process of composite materials. The use of FEM for the structural analysis and characterization of composites offers an insight into its internal behavior in addition to reducing physical testing and its associated costs. However, the reliability of the numerical result is heavily dependent on the adequacy of the input data, with the material parameters of the simple materials playing an important role. Composite manufacturers tend to report the composite properties as a whole rather than specify the component's properties separately [6]. For this reason, correct parameter identification is an issue which is being addressed more and more in this field.

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An optimization method for the determination of material parameters in damaged composite structures is presented in this paper. The proposed methodology faces an inverse problem from a numerical point of view. It adjusts the material parameters of a composite test specimen with unknown properties but available experimental results.

The experimental set-up is reproduced in FEM using the in-house code PLCd [7], with the material properties taking the values assigned by the in-house optimizer Optimate [8]. The numerical result is compared with the experimental data to obtain an error value for the objective function, which is fed back to Optimate. An l_∞–norm is used to estimate the error. Then, by means of a genetic algorithm, Optimate adjusts the material properties until the numerical result is as close as possible to the experimental one.

Several authors have presented inverse methods for the determination of material parameters based on the same fundamental idea as the methodology proposed here. Markiewicz et al. [9] and Geers et al. [10] first used the inverse approach to determine parameters for material models of an aluminum alloy and a glass-fiber reinforced polypropylene composite, respectively. Since then, several variations and improvements on this method have been presented, with different authors putting more focus on particular aspects of the methodology. These include the type of optimization algorithm used [11–13], the objective function defined [14,15] and the material parameters to identify in the context of its applications [11,16–19].

The work presented here provides a novel way of identifying the material parameters which define the components of a composite material. Even with limited experimental data available, the methodology proposed manages to correctly reproduce numerically its behavior. The coupling between the constitutive model and the optimization algorithm is highly flexible and can be easily adapted to different experimental contexts. In addition, the mixing theory used to formulate the constitutive model of the composite is versatile enough to be capable of numerically representing distinct types of composites as long as the simple materials that compose them are correctly characterized.

In the following section, the mixing theory used to numerically model the composite behavior is detailed, including a brief description of the constitutive models considered for the component materials. Section 3 describes the optimization method developed to determine the material parameters of the simple materials which form a composite. An example which illustrates the utility of the coupling between the optimization algorithm and the mixing theory in the proposed methodology is provided in Section 4 to validate the method. Finally, the conclusions of the work are presented.

2. Constitutive modeling

The classical mixing theory assumes strain compatibility as the closing equation [20]:

$$\epsilon_{ij} = (\epsilon_{ij})_1 = (\epsilon_{ij})_2 = \cdots = (\epsilon_{ij})_c \tag{1}$$

where ϵ_{ij} is the strain of the composite material and the subscript $(\bullet)_c$ refers to the c-component of the composite material.

The hypothesis that the contribution of each component is proportional to its volumetric participation is enforced through the specific Helmholtz free energy:

$$\Psi = \sum_{c=1}^{n} k_c \Psi_c; \quad \sum_{c=1}^{n} k_c = 1 \tag{2}$$

where n is the total number of components and k is the volume fraction, which must fulfill the mass conservation principle.

By means of the Clausius-Planck inequality, the secant constitutive equation for the whole composite is obtained in the standard manner [20–22]:

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \epsilon_{ij}} = \sum_{c=1}^{n} k_c \frac{\partial \Psi_c}{\partial \epsilon_{ij}} = \sum_{c=1}^{n} k_c (\sigma_{ij})_c \tag{3}$$

where σ_{ij} is the Cauchy stress tensor. The expression for the free energy of each component Ψ_c will depend on the type of constitutive model chosen for each simple material. In this work, the composite being modeled is a carbon fiber reinforced epoxy matrix, so an anisotropic elasto-plastic constitutive model is proposed for the fibers and an isotropic scalar damage for the matrix [23]. However, other constitutive equations could be easily introduced if required for different type of composites.

2.1. Anisotropic elasto-plasticity

The anisotropic elasto-plastic constitutive model is based on the generalization of the classical plasticity theory [20,24]. The anisotropic theory used to derive this model [23,25,26] is based on the concept of *mapped stress tensor* first introduced by Betten [27].

2.1.1. Plastic damage model

The specific Helmholtz free energy of an elasto-plastic material is:

$$\Psi = \Psi^{e} + \Psi^{p} = \frac{1}{2} \epsilon_{ij}^{e} \mathbb{C}_{ijkl} \epsilon_{kl}^{e} + \Psi^{p} \tag{4}$$

where Ψ^e is the specific elastic free energy, Ψ^p is the specific plastic free energy, $\mathbb{C}_{ijkl}$ is the constitutive tensor of the material and ϵ_{ij}^e is the elastic strain. The total strain is split into an elastic and a plastic part, following the Prandtl–Reus hypothesis:

$$\epsilon_{ij} = \epsilon_{ij}^{e} + \epsilon_{ij}^{p} \tag{5}$$

Then, the constitutive equation of an isotropic elasto-plastic material is:

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \epsilon_{ij}} = \mathbb{C}_{ijkl} (\epsilon_{kl} - \epsilon_{kl}^{p}) \tag{6}$$

The plastic strain is obtained by means of the flow rule:

$$\dot{\epsilon}_{ij}^{p} = \dot{\lambda} \frac{\partial G^{\sigma}}{\partial \sigma_{ij}} \tag{7}$$

where $\dot{\lambda}$ is the plastic consistency factor as derived by Simo and Ju [28] and G^σ is the plastic potential function.

To fully characterize the plastic response, the yield function F^σ must satisfy the yield condition and a plastic hardening law must be defined. In this case, the expression proposed by Oller [29] is used:

$$\dot{\kappa}^{p} = \mathbf{h}_{ij} \dot{\epsilon}_{ij}^{p} \tag{8}$$

where κ^p is the plastic damage internal variable and $\mathbf{h}_{ij}$ is a second-order tensor defined in [29] which requires the definition of a scalar hardening parameter, H.

2.1.2. Anisotropy theory

Anisotropy is modeled by transporting all the constitutive parameters of the material and its stress and strain states from a *real anisotropic space* to a *fictitious isotropic space*. This mapping technique allows reproducing the behavior of the real anisotropic material by means of a well known and developed constitutive model of an isotropic material. The two spaces are related through a linear transformation, using a fourth-order tensor which contains all the information regarding the anisotropy of the real material. It is assumed that both spaces have the same elastic strains, which are related through the *strain transformation tensor* a_{ijkl}^e: