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A discussion on evaluation of material length scale parameter based on micro-cantilever test

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ABSTRACT

Modified couple stress Euler–Bernoulli and constitutive beam models have been used to evaluate material length scale parameter based on experimental data extracted from micro-cantilever test. By comparison study, it is indicated that the Euler–Bernoulli beam model is stiffer than the constitutive beam model and therefore the material length scale parameter obtained based on the constitutive beam model is greater than that obtained based on the Euler–Bernoulli beam model. In addition, a relationship between the aforementioned material length scale parameters is presented.

1. Introduction

Yang et al. [1] modified the couple stress theory by introducing an additional equilibrium relation to govern the behavior of couples and also developed a linear elastic model for isotropic materials in micron scale. The new higher-order elasticity theory employs only one material length scale parameter in addition to Lame constants to capture size effect in micro-structures. Modified couple stress theory has been widely used by many researchers to develop the governing equations of micro-structures and study the size-dependent mechanical behavior of them. Some of these works can be listed as: an Euler–Bernoulli beam model for static bending and vibration analysis by Park and Gao [2] and Kong et al. [3], respectively, a first shear deformation microbeam model for static bending and vibration analysis by Ma et al. [4], a Kirchhoff plate model for static bending analysis by Tsiatas [5], nonlinear static bending, free vibration and post buckling of Euler–Bernoulli microbeams by Xia et al. [6], a functionally graded Euler–Bernoulli microbeam model for static bending and dynamic analysis by Asghari et al. [7], mechanical behavior analysis of a functionally graded microbeam subjected to a thermal moment and nonlinear electrostatic pressure by Mohammadi-Alasti et al. [8], a first shear deformation composite microbeam model based on a new modified couple stress theory which obtains by defining an asymmetric curvature tensor by Chen et al. [9], static bending, buckling and vibration of functionally graded annular microplates by Ke et al. [10], buckling analysis of a functionally graded microbeam resting on two-parameter elastic foundation using a unified higher order beam theories by Şimşek and Reddy [11], free vibration analysis of axially functionally graded tapered Bernoulli–Euler microbeams by Akgöz and Civalek [12], nonlinear free and forced vibration of geometrically imperfect microbeams by Farokhi et al. [13], thermal effect on critical buckling load and natural frequencies of functionally graded Euler–Bernoulli and Timoshenko microbeams by Nateghi and Salamat-talab [14], a Mindlin plate finite element for static bending, buckling and vibration analysis of microplates by Zhang et al. [15], a generalized thermoelasticity model for Timoshenko microbeams by Taati et al. [16], functionally graded Kirchhoff and Mindlin microplate models for nonlinear static bending, stability and vibration analysis by Thai and Choi [17], nonlinear analysis of functionally graded piezoelectric actuator based on Timoshenko beam theory and the von Kármán nonlinearity by Komijani et al. [18], Euler–Bernoulli and Timoshenko beam element for nonlinear static and dynamics analysis of functionally graded microstructure-dependent beams by Arbind and Reddy [19], simulation of fluid–structure interaction in a microchannel by a coupled lattice Boltzmann-finite element approach considering Knudsen number by Esfahanian et al. [20], nonlinear oscilation of functionally graded Mindlin microplates by Ansari et al. [21], free vibration analysis of shear deformable functionally graded cylindrical shell by Tadi Beni et al. [22], modeling of a functionally graded micro-ring segment for the analysis of coupled extensional–flexural waves by Mustapha [23], buckling analysis of micro composite laminated Euler–Bernoulli and Timoshenko microbeams by Mohammad Abadi and Daneshmehr [24], an investigation of

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incorporating the Poisson effect in microbeam models using available experimental data of an epoxy micro-cantilever by Dehrouyeh-Semnani and Nikkhah-Bahrami [25], dynamic characteristics of Euler–Bernoulli microbeams considering micro-inertia effect by Fathalilou et al. [26], nonlinear modeling of curved microtubes conveying fluid for vibration analysis by Tang et al. [27], thermo-mechanical buckling behavior of functionally graded microbeams embedded in elastic medium by Akgöz and Civalek [28], nonlinear static and free vibration analysis of microbeams resting on nonlinear elastic foundation by Şimşek [29], nonlinear forced vibration of microplates by Ghayesh et al. [30], modified version of couple stress theory in general curvilinear coordinates by Ashoori and Mahmoodi [31] and dependency of material length scale parameters to higher-order continuum theory unlike Lame constants by Dehrouyeh-Semnani [32].

Modified couple stress beam models have been employed to analyze experimental data extracted from micro-cantilever tests. Park and Gao [2] employed experimental bending rigidity of an epoxy micro-cantilever reported by Lam et al. [33] to validate the modified couple stress Euler–Bernoulli beam model. They reported that the beam results agree fairly well with the experimental data. Moreover, they reported the value of material length scale parameter of epoxy based on the constitutive beam model is 17.6 μm. Dehrouyeh-Semnani [34] analyzed the experimental results of the epoxy micro-cantilever proposed by Lam et al. [33] by using the constitutive beam model and showed that the constitutive beam model validates very well the experimental data on the basis of material length scale parameter reported by Park and Gao [2]. Rahaeifard et al. investigated static pull-in phenomena in micro-cantilever based on modified couple stress theory. They used modified couple stress Euler–Bernoulli beam model proposed by Park and Gao [2] to analyze experimental results reported by Osterberg [35]. Based on their work, the material length scale parameter of silicon is 0.592 μm. Baghani [36] presented analytical solution for size-dependent static pull-in voltage of micro-cantilever based on modified couple stress Euler–Bernoulli beam model. By using the experimental data reported by Osterberg [35], he showed the material length scale parameter of silicon equals 0.6125 μm. In addition, by considering uncertainties in the electrostatically actuated micro-cantilever, the aforementioned experimental data was analyzed by Rokni et al. [37] on the basis of modified couple stress Euler–Bernoulli beam model. Kahrobaiyan et al. [38] proposed a yield criterion model based on modified couple stress theory. By employing the values of yield strength of Aluminum micro-cantilever reported by Son et al. [39] and modified couple stress Euler–Bernoulli beam model, they showed the material length scale parameter of Aluminum is equal to 0.35 μm. McFarland and Colten [40] conducted a bending test on a Polypropylene micro-cantilever. They employed modified couple stress constitutive beam model to analyze the obtained experimental data. Based on their work, the higher-order parameter of Polypropylene is equal to 32.0 μm and 53.79 μm when the thickness of micro-cantilever is equal to 15.85 and 29.37, respectively. Assuming the Poisson ratio of Polypropylene is about 0.47, the material length scale parameter of Polypropylene can be obtained as follows: 25.38 μm for t = 15.85 μm and 42.66 μm for t = 29.37 μm which t is the thickness of the micro-cantilever. It can be easily concluded that the modified couple stress theory is incapable of predicting size effect in the Polypropylene micro-cantilever using one constant material length scale parameter.

Difference between modified couple stress Euler–Bernoulli and constitutive beam models leads to evaluation of different values of material length scale parameter based on micro-cantilever test. Therefore, performing a comparison study between the aforementioned size-dependent beam models seems to be essential. In this study, the stiffness bending parameter of a micro-cantilever based on the aforementioned beam models is compared and also a relationship is derived between the material length scale parameters based on the aforementioned beam models for both plane stress and plane strain conditions.

2. Modified couple stress micro-cantilever models

Lam et al. [33] proposed modified strain gradient elasticity beam model based on constitutive relations. Modified strain gradient elasticity theory employs three length scale parameter (i.e., ℓ_0, ℓ_1 and ℓ_2) to capture size effect. By letting $\ell_0 = \ell_1 = 0$ and $\ell_2 = \ell$ in modified strain gradient elasticity theory, modified couple stress theory is obtained [34]. Hence, the constitutive microbeam model based on modified couple stress theory can be obtained by [34,40]

$$D_c \frac{d^4 w}{dx^4} = -q \tag{1}$$

where

$$D_c = D_0\left(1 + \left(\frac{b_h}{h}\right)^2\right) \tag{2}$$

where w, q, h, b_h and D_0 stand for static deflection of the microbeam, distributed load on the microbeam, thickness of the microbeam, higher-order bending parameter which characterize the thickness dependence of beam bending and conventional bending parameter, respectively.

$$b_h^2 = 3(1-\nu)\ell^2, \quad D_0 = \frac{Ebh^3}{12\phi} \tag{3}$$

where ν, ℓ and b stand for Poisson ratio, material length scale parameter and width of the microbeam. In addition, when plane stress condition governs $\phi = 1$ and when plane strain condition governs $\phi = 1 - \nu^2$.

The associated boundary conditions can be obtained as follows [34]

$$\begin{aligned} &D_c \frac{\partial^3 w}{\partial x^3} = \overline{Q} \quad \text{or} \quad w = \bar{w} \\ &D_c \frac{\partial^2 w}{\partial x^2} = -\overline{M} \quad \text{or} \quad w' = \bar{w}' \end{aligned} \tag{4}$$

All quantities with an over bar indicate specified values. The governing equation of modified couple stress Euler–Bernoulli can be obtained by [2,34]

$$D_{EB} \frac{d^4 w}{dx^4} = -q \tag{5}$$

where D_{EB} can be obtained by (see Eq. 22 in [2]):

$$D_{EB} = D_0\left[1 + \frac{6\phi}{1+\nu}\left(\frac{\ell}{h}\right)^2\right] \tag{6}$$

It is notable that in Ref. [34] the formulation of bending rigidity of Euler–Bernoulli beam model for plane-stress condition was written as plane-stress condition, however the reported results in the aforementioned reference were obtained based on plane-strain condition. The associated boundary conditions can be obtained as follows:

$$\begin{aligned} &D_{EB} \frac{\partial^3 w}{\partial x^3} = \overline{Q} \quad \text{or} \quad w = \bar{w} \\ &D_{EB} \frac{\partial^2 w}{\partial x^2} = -\overline{M} \quad \text{or} \quad w' = \bar{w}' \end{aligned} \tag{7}$$

3. Comparison study

In this section, stiffness bending parameter ratio of modified couple stress Euler–Bernoulli and constitutive beam models are