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Free vibration of anti-symmetric angle-ply laminated conical shells

K.K. Viswanathan [a,*], Saira Javed [a], Kandasamy Prabakar [b], Z.A. Aziz [a], Izliana Abu Bakar [a]

[a] UTM Centre for Industrial and Applied Mathematics, Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Skudai, Johor Bahru, Johor, Malaysia

[b] School of Electrical Engineering, Pusan National University, San 30, Jangjeong-Dong, Gumjeong-Ku, Busan 609735, South Korea

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ABSTRACT

Free vibration of anti-symmetric angle-ply composite laminated conical shells is studied including shear deformation using spline function approximation. The equilibrium equations are formulated in terms of displacement and rotational functions. These functions are approximated using Bickley-type splines to obtain the generalised eigenvalue problem with suitable boundary conditions. Parametric studies are made to analyse the effects of circumferential node number, length ratio and cone angle on the frequency parameter for different number of layers and materials with different ply orientations under two types of boundary conditions.

1. Introduction

The shell structures offers the major contribution in the field of civil engineering, architectural, aeronautical, structural and marine engineering. The extensive use of the shell structures in engineering is due to the desirable characteristics such as higher specific stiffness, better damping and shock absorbing. They can carry applied loads effectively by means of their curvature, although the geometry of shells is thin, light and span over the large areas.

Shu [1,2] presented an efficient approach for the free vibration analysis of conical shells using the method of generalised differential quadrature (GDQ). Wu and Lee [3] analysed the free vibration of laminated conical shells with variable stiffness applying the method of differential quadrature (DQ). The Rayleigh–Ritz method was used by Hu et al. [4] to examine the vibration of twisted laminated composite conical shells. Correia et al. [5] used higher order shear deformation theory to analyse the laminated conical structures. Qatu [6] made a comprehensive study about vibration of laminated shells and plates. A numerical study on the free vibration analysis for laminated conical and cylindrical shells is presented by Civalek [7]. The transfer matrix method was used by Liang et al. [8] to investigate the natural vibration of a symmetric cross-ply laminated composite conical-plate shell. Qatu et al. [9] investigated the recent research advances on the dynamic analysis of composite shells

The non-homgeneous orthotropic composite truncated conical shells were analysed for large-amplitude vibration using Superposition principle, Galerkin and semi-inverse methods by Sofiyev [10]. Moreover, Sofiyev and Kuruoglu [11] analysed symmetric and anti-symmetric cross-ply laminated cylindrical shells using classical shell theory and shear deformation theory. Najafov et al. [12] studied non-linear free vibration of symmetric and anti-symmetric cross-ply laminated truncated conical shells using Galerkin's method. Free vibration of laminated layered cylindrical and symmetric angle-ply laminated cylindrical shells of variable thickness under both classical theory and including shear deformation theory using spline was studied by Viswanathan et al. [13–16].

In this research, free vibration of anti-symmetric angle-ply laminated conical shells made of layers with different materials is studied under shear deformation theory. The problem is formulated using first order shear deformation theory (FSDT) to obtain the equilibrium equations of conical shell. The governing differential equations are derived in terms of displacement and rotational functions using stress–strain and strain–displacement relations in the equilibrium equations. The solution in a separable form is assumed to obtain a system of coupled differential equation, in the longitudinal, circumferential and transverse displacement functions and rotational functions which are approximated by cubic spline. Collocation with these splines yields a set of field equations which, along with the equation of boundary conditions, reduces to a system of homogeneous equations on the assumed spline coefficients to obtain a generalised eigenvalue problem. This eigenvalue problem is solved to obtain eigenfrequency parameter. The variation of the frequency parameter with respect to cone angle, aspect ratio, circumferential node number, boundary conditions, and two types of layered materials of four layered conical shells are analysed. In this work, the problem is solved numerically by using spline approximation.

* Corresponding author.
E-mail address: visu20@yahoo.com (K.K. Viswanathan).

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2. Theoretical formulation and method of solution

2.1. Formulation of the problem

Consider a composite laminated conical shell as shown in Fig. 1 with an arbitrary number of layers. Based on Fig. 1, r_a and r_b are the radii of the cone at its small and large edges, α is a semi vertex angle of the cone, ℓ is the length of the cone along its generator and h is the wall thickness. The cone's radius at any point along its length is $r = x\sin\alpha$. The radius at small edge is $r_a = a\sin\alpha$ while at large edge, $r_b = b\sin\alpha$. The conical shells is referred to a coordinate system (x,θ,z) is fixed at its reference (which is taken to be at the middle surface) as shown in Fig. 1. The coordinate system, x is measured along the cone's generator starting at the middle length, θ is the circumferential coordinate and z is the normal to the surface (outward positive). The displacements of the shell's middle surface are denoted by U, V and W along x, θ and z directions, respectively. In addition, by considering shear deformation in this study, there is rotations of the normal are denoted by ψ_x and ψ_θ along x and θ axes respectively.

The equations of equilibrium for truncated conical shells are

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{1}{x}(N_x - N_\theta) + \frac{1}{x\sin\alpha}\frac{\partial N_{x\theta}}{\partial\theta} = \rho h \frac{\partial^2 u}{\partial t^2}\\
&\frac{\partial N_{x\theta}}{\partial u^2} + \frac{2}{x}N_{x\theta} + \frac{1}{x\sin\alpha}\frac{\partial N_\theta}{\partial\theta} + \frac{1}{x\tan\alpha}Q_\theta = \rho h \frac{\partial^2 v}{\partial t^2}\\
&\frac{\partial Q_x}{\partial x} + \frac{1}{x\sin\alpha}\frac{\partial Q_\theta}{\partial\theta} + \frac{1}{x}Q_x - \frac{1}{x\tan\alpha}N_\theta = \rho h \frac{\partial^2 w}{\partial t^2}\\
&\frac{\partial M_x}{\partial x} + \frac{1}{x}(M_x - M_\theta) + \frac{1}{x\sin\alpha}\frac{\partial M_{x\theta}}{\partial\theta} - Q_x = \frac{\rho h^3}{12}\frac{\partial^2 \psi_x}{\partial t^2}\\
&\frac{\partial M_{x\theta}}{\partial x} + \frac{2}{x}M_{x\theta} + \frac{1}{x\sin\alpha}\frac{\partial M_\theta}{\partial\theta} - Q_\theta = \frac{\rho h^3}{12}\frac{\partial^2 \psi_\theta}{\partial t^2}
\end{aligned} \tag{1}$$

According to the first order shear deformation theory, the displacements components u, v, w can be written as

$$\begin{aligned}
u(x,\theta,z,t) &= u_\circ(x,\theta,t) + z\psi_x(x,\theta,t)\\
v(x,\theta,z,t) &= v_\circ(x,\theta,t) + z\psi_\theta(x,\theta,t)\\
w(x,\theta,z,t) &= w(x,\theta,t)
\end{aligned} \tag{2}$$

where u_0, v_0, w_0 are the mid plane displacements, and ψ_x, ψ_θ are the shear rotations of any point on the mid surface normal to the xz and θz plane respectively and t is the time.

The strain–displacement relations for cylindrical shells having the radius r given as

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x}\\
\varepsilon_\theta &= \frac{1}{x}u + \frac{1}{x\tan\alpha}w + \frac{1}{x\sin\alpha}\frac{\partial v}{\partial\theta}\\
\nu_{x\theta} &= \frac{1}{x\sin\alpha}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial x} - \frac{v}{x}\\
\kappa_x &= \frac{\partial\psi_x}{\partial x}\\
\kappa_\theta &= \frac{1}{x\sin\alpha}\frac{\partial\psi_\theta}{\partial\theta} + \frac{\psi_x}{x}\\
\kappa_{x\theta} &= \frac{\partial\psi_\theta}{\partial\theta} - \frac{\psi_\theta}{x} + \frac{1}{x\sin\alpha}\frac{\partial\psi_x}{\partial\theta}
\end{aligned} \tag{3}$$

The stress–strain relations of the kth layer by neglecting the transverse normal strain and stress, are of the form

$$\{\sigma\} = \left[Q^{(k)}\right]\left\{\varepsilon_x^{(k)}\varepsilon_\theta^{(k)}\gamma_{x\theta}^{(k)}\gamma_{xz}^{(k)}\gamma_{\theta z}^{(k)}\right\} \tag{4}$$

When the materials are oriented at an angle θ with the x-axis, the transformed stress–strain relations are

$$\{\sigma\} = \left[\overline{Q}^{(k)}\right]\left\{\varepsilon_x^{(k)}\varepsilon_\theta^{(k)}\gamma_{x\theta}^{(k)}\gamma_{xz}^{(k)}\gamma_{\theta z}^{(k)}\right\} \tag{5}$$

where $Q_{ij}^{(k)}$ and $\overline{Q}_{ij}^{(k)}$ are given in Viswanathan et al. [17].

Where the stress resultants and stress couples are given

$$\begin{aligned}
(N_x, N_\theta, N_{x\theta}, Q_x, Q_\theta) &= \int_z (\sigma_x, \sigma_\theta, \tau_{x\theta}, \tau_{xz}, \tau_{\theta z})dz\\
(M_x, M_\theta, M_{x\theta}) &= \int_z (\sigma_x, \sigma_\theta, \tau_{x\theta})zdz
\end{aligned} \tag{6}$$

Applying the Eq. (3) into Eq. (5) and then substituting into the Eq. (6), to obtain the equations of stress-resultants and moment resultant as

$$\begin{pmatrix} N_x\\ N_\theta\\ N_{x\theta}\\ M_x\\ M_\theta\\ M_{x\theta} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16}\\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26}\\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66}\\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16}\\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26}\\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_x\\ \varepsilon_\theta\\ \nu_{x\theta}\\ \kappa_x\\ \kappa_\theta\\ \kappa_{x\theta} \end{pmatrix}$$

and

$$\begin{pmatrix} Q_x\\ Q_\theta \end{pmatrix} = K\begin{pmatrix} A_{45} & A_{55}\\ A_{44} & A_{45} \end{pmatrix}\begin{pmatrix} \varepsilon_{\theta z}\\ \varepsilon_{xz} \end{pmatrix} \tag{7}$$

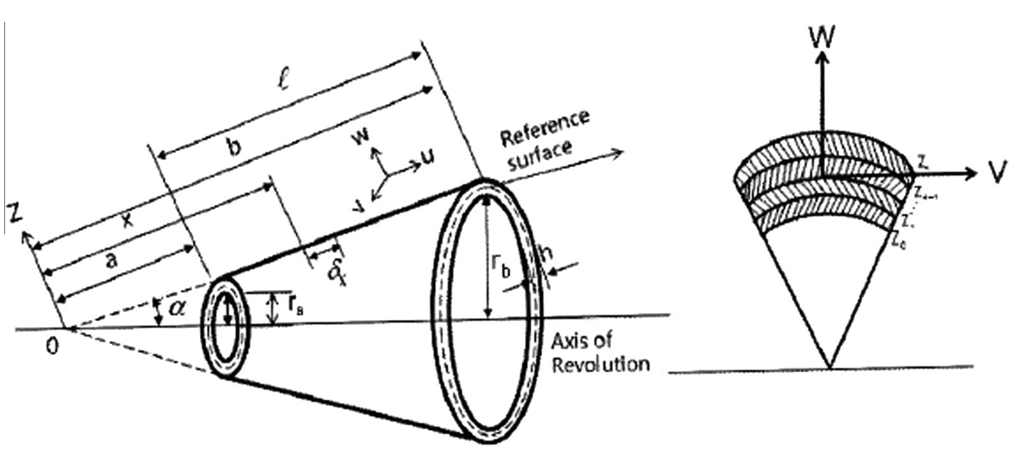

Fig. 1. Layered conical shell.