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Stochastic free vibration analysis of angle-ply composite plates – A RS-HDMR approach

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ABSTRACT

This paper presents a generic random sampling-high dimensional model representations (RS-HDMR) approach for free vibration analysis of angle-ply composite plates. A metamodel is developed to express stochastic natural frequencies of the system. A global sensitivity analysis is carried out to address the influence of input random parameters on output natural frequencies. Three different types of input variables (fiber-orientation angle, elastic modulus and mass density) are varied to validate the proposed algorithm. The present approach is efficiently employed to reduce the sampling effort and computational cost when large number of input parameters is involved. The stochastic finite element approach is coupled with rotary inertia and transverse shear deformation based on Mindlin's theory. Statistical analysis is carried out to illustrate the features of the RS-HDMR and to compare its performance with full-scale Monte Carlo simulation results. The stochastic mode shapes are also depicted for a typical laminate configuration. Based on the numerical results, some new physical insights are drawn on the dynamic behavior of the system.

1. Introduction

Composite materials are extensively used for small components as well as large structures such as airplanes, ships, wind turbine blades and small bridges due to its weight sensitivity, cost-effective, high specific stiffness and can handle different strength at different directions. There has been an increasing demand for composite materials especially in aircraft industries. Because of its inherent complexity, laminated composite structures can be difficult to manufacture accurately according to its exact design specifications, resulting in undesirable uncertainties. The random structural uncertainties involve material properties, fiber parameters of the individual constituent laminae. Because of the randomness in geometry and material properties of laminated composite structures, the mass and stiffness matrices become stochastic in nature. The composite materials are generally made of fiber and matrix through an appropriate fabrication process in manufacturing with three basic steps namely tape, layup and curing. Since the mechanical properties of constituent material vary statistically, the source of uncertainties in composite material properties are produced from randomness in material properties as well as from uncertainties in the fabrication parameters. The uncertainties incurred during the layup process are due to the misalignment of ply-orientation. Typical uncertainties incurred from the curing process are intralaminate voids, incomplete curing of resin, excess resin between plies, excess matrix voids and porosity and variation in ply thickness. These variables are statistical in nature; therefore, the properties of composite materials should be quantified probabilistically. As a consequence, the behavior of composite structures shows a scatter from its average value. Traditionally, an ad hoc factor of safety is used in the design to account for the difficulty in predicting the structural behavior. However, this approach of designer may result in either an ultraconservative or an unsafe design.

In order to probabilistically assess the behavior of composite structures, these uncertainties are considered due to random variation of material properties and ply parameters. These uncertainties are referred to as the random variables which can be processed computationally through composite mechanics and probability method. Due to random input parameters, the uncertainties in the specification of mass and stiffness matrices induce the statistical variation in eigenvalues and eigenvectors, resulting in subsequently fluctuation of natural frequencies. Therefore a realistic analysis of composite laminated plates is required to arrest the volatility of natural frequencies arising from the randomness in the variation of parameters like ply-orientation angle, elastic modulus and mass density. In general, uncertainties can be

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categorized into three types, namely aleatoric, epistemic and prejudicial, respectively. The first type of uncertainty is aleatoric uncertainty which is due to the inherent variability in the system parameters, for example, different cars manufactured from a single production line are not exactly the same. If enough samples are present, it is possible to characterize the variability using well established statistical methods and the probably density functions of the parameters can be obtained. The second type of uncertainty is epistemic uncertainty which is due to lack of knowledge regarding a system. This type of uncertainty is generally arisen in the modeling of complex systems, for example the problem of predicting cabin noise in helicopters. Unlike aleatoric uncertainties, it is recognized that probabilistic models are not quite suitable for epistemic uncertainties. The third type of uncertainty is prejudicial uncertainty which is similar to the first type except that the corresponding variability characterization is not available, in which case work can be directed to gain better knowledge. An example of this type of uncertainty is the use of viscous damping model in spite of knowing that the true damping model is not viscous. The total uncertainty of a system is the combination of these three types of uncertainties.

In general, Monte Carlo simulation technique is popularly utilized to generate the randomized output frequency to deal with large number of samples. Although the uncertainty in material and geometric properties can be computed by the MCS method, it is inefficient and expensive. To mitigate this lacuna, random sampling – high-dimensional model representations (RS-HDMR) is employed for quantitative model assessment and analysis tool which maps high-dimensional input–output system relationship very efficiently [1]. In this problem the actual finite element model is replaced by a response surface metamodel, making the process computationally efficient and cost effective. When input variables are randomly sampled for constructing the metamodel, a random sampling – HDMR can be constructed [2,3]. Over the years, HDMR is successfully applied in many different fields [4–6]. The effect of material uncertainty on vibration control of smart composite plate is studied by Umesh and Ganguli [7] by using polynomial chaos expansion. The dynamic stability of uncertain laminated beams subjected to subtangential loads studied by Goyal and Kapania [8] while Manan and Cooper [9] introduced probabilistic approach for design of composite wings including uncertainties. On the other hand, Fang and Springer [10] studied on the design of composite laminates by a Monte Carlo method while Kapania and Goyal [11] investigated on free vibration of unsymmetrically laminated beams. The stochastic eigenvalue problem is studied with polynomial chaos [12] and Karhunen–Loeve expansion of non-gaussian random process [13]. Of late, Chowdhury and Adhikari [14] investigated on high dimensional model representation for stochastic finite element analysis while Talha and Singh [15] investigated on stochastic perturbation-based finite element for buckling statistics of functionally graded plates with uncertain material properties in thermal environments.

In the present analysis, random samples are drawn uniformly over the entire domain ensuring good prediction capability of the constructed metamodel in the whole design space including the tail regions. The present work aims to develop an algorithm for uncertainty quantification of natural frequencies of cantilever composite plate using RS-HDMR and its comparative efficacy compared to direct Monte Carlo simulation (MCS) technique. The sensitivity analysis is carried out to cross-validate the results for constructing the metamodel and subsequently the number of sample runs required to fit the constructed metamodel is catastrophically reduced. An eight noded isoparametric plate bending element with five degrees of freedom at each node is considered in finite element formulation to study the randomized natural frequencies (see Fig. 1).

2. Governing equations

The laminated composite cantilever plate is considered with uniform thickness with the principal material axes of each layer being arbitrarily oriented with respect to mid-plane. If the mid-plane forms the x–y plane of the reference plane, then the displacements can be computed as

$$\begin{aligned} u(x,y,z) &= u^0(x,y) - z\theta_x(x,y) \\ v(x,y,z) &= v^0(x,y) - z\theta_y(x,y) \\ w(x,y,z) &= w^0(x,y) = w(x,y), \end{aligned} \tag{1}$$

where u, v and w are the displacement components in x-, y- and z-directions, respectively and u^0, v^0 and w^0 are the mid-plane displacements, and θ_x and θ_y are rotations of cross-sections along the x- and y-axes. In general, the force and moment resultants of a single lamina are obtained from stresses as

$$\begin{aligned} \{F\} &= \{N_x\ N_y\ N_{xy}\ M_x\ M_y\ M_{xy}\ Q_x\ Q_y\}^T \\ &= \int_{-t/2}^{t/2} \{\sigma_x\ \sigma_y\ \tau_{xy}\ \sigma_x z\ \sigma_y z\ \tau_{xy} z\ \tau_{xz}\ \tau_{yz}\}^T dz \end{aligned} \tag{2}$$

In matrix form, the in-plane stress resultant {N}, the moment resultant {M}, and the transverse shear resultants {Q} can be expressed as

$$\{N\} = [A]\{\varepsilon^0\} + [B]\{k\} \quad \text{and} \quad \{M\} = [B]\{\varepsilon^0\} + [D]\{k\} \tag{3}$$

$$\{Q\} = [A^*]\{\gamma\} \tag{4}$$

where

$$\left[A^*_{ij}\right] = \int_{-t/2}^{t/2} \overline{Q}_{ij} dz \quad \text{where} \quad i,j = 4,5$$

$$[\overline{Q}_{ij}(\varpi)] = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & (m^4+n^4) & -4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2) \\ m^3n & mn^3 & (mn^3-m^3n) & 2(mn^3-m^3n) \\ mn^3 & m^3n & (m^3n-mn^3) & 2(m^3n-mn^3) \end{bmatrix} [Q_{ij}]$$

Here $m = Sin\theta(\overline{\omega})$ and $n = Cos\theta(\overline{\omega})$, wherein $\theta(\overline{\omega})$ is the random fiber orientation angle. However, laminate consists of a number of laminae wherein $[Q_{ij}]$ and $[\overline{Q}_{ij}(\overline{\omega})]$ denotes the On-axis elastic constant matrix and the off-axis elastic constant matrix, respectively. The elasticity matrix of the laminated composite plate is given by,

$$[D'(\varpi)] = \begin{bmatrix} A_{ij}(\varpi) & B_{ij}(\varpi) & 0 \\ B_{ij}(\varpi) & D_{ij}(\varpi) & 0 \\ 0 & 0 & S_{ij}(\varpi) \end{bmatrix} \tag{5}$$

where

$$[A_{ij}(\varpi), B_{ij}(\varpi), D_{ij}(\varpi)] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\overline{Q}_{ij}(\varpi)][1, z, z^2] dz \quad i,j = 1,2,6 \tag{6}$$

$$[S_{ij}(\varpi)] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \alpha_s [\overline{Q}_{ij}(\varpi)]_k dz \quad i,j = 4,5 \tag{7}$$

where α_s is the shear correction factor and is assumed as 5/6. The mass per unit area is expressed as

$$P(\varpi) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho(\varpi) dz \tag{8}$$

Mass matrix is expressed as

$$[M(\varpi)] = \int_{Vol} [N][P(\varpi)][N] d(vol) \tag{9}$$