Composite Structures 115 (2014) 12–28

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A high-fidelity first-order reliability analysis for shear deformable laminated composite plates

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ARTICLE INFO

Article history:
Available online 4 May 2014

Keywords:
Composite plates
Reliability analysis
Probabilistic analysis
FORM
Monte Carlo simulation

ABSTRACT

This paper presents the complete theoretical development and practical implementation of a first-order reliability analysis for shear deformable laminated composite plates. The choice of plate theory is initially presented as it is important to ensure that a formulation capable of representing realistic physics is used as the basis of the overall simulation tool to reduce epistemic uncertainty as far as possible. The first-order reliability method (FORM) is proposed for the reliability analysis and summarised in the paper, with comparisons made throughout the paper with Monte Carlo simulation. The sensitivities required as part of the FORM algorithm are presented and verified against finite difference approximations. The practical implementation of the computational framework is demonstrated by numerical examples in which the probability failure of plates is calculated with performance criteria based on deflection and stress and uncertainties associated with fibre orientation and ply thickness.

1. Introduction

The growth in application of composite materials reflects the increasing importance of being able to design material properties consistent with mechanics performance metrics. Simulation is a key component of the engineering design process, at both material and component levels, and normally requires experimental validation. It is in this final stage that many deterministic studies have failed to simulate the mechanical behaviour of composite materials and components, with considerable observed differences between theoretical predictions and experimental measurements [1]. Variations in fibre volume fractions, matrix-fibre voids, damage, fibre misalignment, residual stresses, etc. (e.g. [2–5]) introduce uncertainty at a local level that then propagates to a larger scale and is reflected in the variability of stiffness and strength descriptors characterising material or component scale structural performance. In addition to the aleatoric uncertainty representing natural or intrinsic variability, epistemic uncertainty describes knowledge or information that is missing because, for example, quantities may not have been measured or may have been measured with insufficient accuracy, loading and boundary conditions have been inadequately represented, the numerical representation contains assumptions that results in certain phenomena being omitted or misrepresented, or the analysis method at both simulation and reliability levels are inappropriate or inadequate [6,7]. The significance of these uncertainties is reflected in the use of high safety factors in deterministic structural analysis, and is particularly manifested as engineering conservatism in the presence of modelling or simulation uncertainty.

The complexity of laminated composites is partly reflected in the different approaches that are available to study these materials and structures. Single layer and discrete layer theories have been proposed in which the laminated structure is treated respectively as either a type a homogenised whole or a combination of individual layers (layer-wise). Plate theories are similarly divided into stress-based and displacement-based theories. Shear deformation may also contribute significantly to the behaviour of a composite plate or shell. Shear deformation theories are often considered to be those that are represented in an equivalent single layer formulation, whereas the theories that are "layer-wise" are not normally included in this category, even though shear effects are considered in these models. Transverse shear stress components are absent in classical laminated plate theory which may lead to errors for thick plates, especially where the transverse shear stiffness is low, as often found for advanced composites, making the inclusion of shear deformation a normal prerequisite for the analysis of composite plate and shell structures. We consider an equivalent single layer formulation in this paper (the sine approach of Touratier) in contrast to a layer-wise approach.

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http://dx.doi.org/10.1016/j.compstruct.2014.04.007
0263-8223/

The reliability analysis of composites is challenging because it combines uncertainty quantification with the numerical estimation of behaviour and performance criteria that are themselves complex. Fundamental reliability analysis techniques have been developed and applied to a number of fields. The need for a framework into which to set this work has been identified [7]. A required component of this framework will be the identification of appropriate solid mechanics formulations that inherently minimise the epistemic uncertainty associated with the simulation. Recent research demonstrates the combination of reliability analysis with classical lamination theory [9], Mindlin theory [8], and higher-order (cubic) shear deformation theory [10]. In this paper we present a high-fidelity first-order reliability analysis for laminated composite plates.

2. Plate formulation

2.1. Selection

It is well known that with a ratio of elastic modulus to shear modulus of the order of 25–40, compared with a value of 2.6 for a typical isotropic material, classical Kirchhoff theory (CLT) is unable to simulate the behaviour of an advanced composite plate. First-order shear deformation plate theory (FSDT) assumes transverse shear strains that are, along with the through-thickness shear stresses, constant through the thickness, contradicting the physical behaviour. Whilst the shear stresses cannot be corrected within the limitations of the first-order shear deformation plate theory, corresponding shear forces may be modified by a shear correction factor. Therefore, research is always active on the development of new theoretical models for heterogeneous structures. In this context, two families can be identified: the Equivalent Single Layer Models (ESLM), where CLT, FSDT or high-order theories can be found and the Layer-Wise Models (LWM). According to [11], the number of unknowns remains independent of the number of constitutive layers in the ESLM, while the same set of variables is used in each layer for the LWM. Alternatively, new models may be formulated by introducing interface conditions into high-order ESLM or LWM models. This enables the number of unknowns to be reduced and can be viewed as a ZigZag model. Excellent reviews have been made in the following articles [12–17] or in the more recent review [18].

Third-order shear deformation theories assume a quadratic shear stress distribution through the plate thickness (Fig. 1(c)). Since the shear stresses vanish at the upper and lower surfaces of the plate the shear correction factor is no longer needed.

In this work, an efficient theory is needed but LWM is computationally too expensive. A ZigZag model can be viewed as a good compromise between cost and accuracy, as continuity at the layer interfaces and top and bottom free conditions are fulfilled while the number of unknowns remain independent of the number of layers.

Touratier [19] proposed kinematics (referred to as the sine model) which assumed a shear strain distribution through the thickness of the plate in the form of a cosine function as;

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_o(x,y)}{\partial x} + f(z)\gamma_x^0(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_o(x,y)}{\partial y} + f(z)\gamma_y^0(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where $f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right), \gamma_x^0(x,y) = \theta_x + \frac{\partial w}{\partial x}$, and $\gamma_y^0(x,y) = \theta_y + \frac{\partial w}{\partial y}$. With the existence of interlaminar stresses at geometric boundaries such as free-edges, cut-outs, notches, and holes of structural components made of composite laminates are important phenomena [20], Beakou and Touratier [21] developed displacement fields with the inclusion of interlayer continuity, such that;

$$\begin{aligned} u^{(k)}(x,y,z) &= u_0(x,y) - z\frac{\partial w(x,y)}{\partial x} + \left(f_1(z) + g_1^{(k)}(z)\right)\left(\frac{\partial w}{\partial x} + \theta_1\right) \\ &\quad + g_2^{(k)}(z)\left(\frac{\partial w}{\partial y} - \theta_1\right) \\ v^{(k)}(x,y,z) &= v_0(x,y) - z\frac{\partial w(x,y)}{\partial y} + g_3^{(k)}(z)\left(\frac{\partial w}{\partial x} + \theta_2\right) \\ &\quad + \left(f_2(z) + g_4^{(k)}(z)\right)\left(\frac{\partial w}{\partial y} - \theta_1\right) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

where f_1, f_2, and $g_1^{(k)} - g_4^{(k)}$ are:

$$\begin{aligned} f_1(z) &= f(z) - \frac{e}{\pi}b_{55}f'(z) \\ f_2(z) &= f(z) - \frac{e}{\pi}b_{44}f'(z) \\ g_i^{(k)}(z) &= a_i^{(k)}z + d_i^{(k)} \end{aligned} \tag{3}$$

(k) is the layer number, h the thickness of the plate, and coefficients $b_{44}, b_{55}, a_i^{(k)}, d_i^{(k)}$ are determined from the boundary conditions on the top and bottom surfaces and from the continuity requirements at the layer interfaces for displacements and stresses.

For clarity, dropping the superscript (k), Eq. (3) may be interpreted and used to represent existing general types of displacement field. For example, assuming:

1. $f_1(z) = f_2(z) = f(z), g_i^{(k)}(z) = 0$, the Touratier sine model without interlayer continuity is obtained,
2. $f_1(z) = f_2(z) = z, g_i^{(k)}(z) = 0$, produces the Reissner–Mindlin model with shear correction factor,
3. $f_1(z) = f_2(z) = 0, g_i^{(k)}(z) = 0$, is the Kirchhoff–Love model,
4. $f_1(z) = f_2(z) = f(z) = z\left(1 - \frac{4z^2}{3e^2}\right), g_i^{(k)}(z) = 0$, describes other higher order models.

The Touratier sine model kinematic, using five unknown functions as FSDT, has been selected for the research presented in this paper. It produces results that represent improvements over polynomial type models for thin laminated plate analysis. Polit and Touratier [22] presented a finite element (FE) implementation of this theory for semi-thick to thin composite plate modelling. It exhibited high convergence rates and accuracies for both displacements and stresses. It was shown [22] that the sine model produced an excellent approximation to the analytical analysis of Pagano [23] compared to other theories with the same complexity. This maybe explained by the fact that the series of sine function is very much richer than a polynomial function, because the sine function can be formed by an infinite series as:

$$f(z) = \frac{h}{z}\sin\frac{\pi z}{h} = \frac{h}{z}\left(\sum_{p=0}^{\infty}(-1)^p\left(\frac{\pi z}{h}\right)^{2p+1} \Big/ (2p+1)!\right) \tag{4}$$

It has been implemented using a discretisation comprising six node triangles based on the approximations of Argyris et al. [24] and Ganev and Dimitrov [25]. Argyris's FE is a fifth order polynomial interpolation function used for the deflection, while a fourth order polynomial interpolation is used with additional rotation and membrane displacement unknowns. Furthermore, the combined Argyris–Ganev FE displays no spurious energy modes or shear locking [22].