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Bending and free vibration of functionally graded piezoelectric beam based on modified strain gradient theory

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ABSTRACT

A size-dependent functionally graded piezoelectric beam model is developed using a variational formulation. It is based on the modified strain gradient theory and Timoshenko beam theory. The material properties of functionally graded piezoelectric beam are assumed to vary through the thickness according to a power law. The new model contains three material length scale parameters and can capture the size effect, unlike the classical beam theory. To illustrate the new functionally graded piezoelectric beam model, the static bending and free vibration problems of a simply supported beam are numerical solved. These results may be useful in the analysis and design of smart structures constructed from piezoelectric materials.

1. Introduction

Piezoelectric materials have been widely used as sensors and actuators in control systems due to their excellent electromechanical properties, design flexibility, and efficiency to convert electrical energy into mechanical energy. Traditional piezoelectric sensors and actuators are often made of several layers of different piezoelectric materials. Many theoretical and mathematical models have been presented for laminated composite structures with piezoelectric sensors and actuators [1–4]. The principal weakness of these structures is that the high stress concentrations are usually appeared at the layer interfaces under mechanical or electrical loading. This drawback restricts the usefulness of piezoelectric devices in the areas where the devices require high reliability.

In order to overcome the performance limitations of the traditional layered piezoelectric elements, the concept of functionally graded materials (FGMs) has been extended into the piezoelectric materials by recent advances in the metallurgical science and fabrication techniques [5]. Due to continuous change in the material composition and properties, these kinds of advanced materials are called functionally graded piezoelectric materials (FGPMs).

With the developments in nanotechnology, microbeam, in which its thickness is generally on the order of microns and sub-microns, has been extensively used in many applications of micro- and nano-size devices and systems, such as microsensors [6], microactuators [7], nano- and micro-electromechanical systems (NEMS and MEMS) [8]. In such applications, size effects or small scale effects are experimentally observed [9–12]. Conventional continuum models based on classical continuum theories do not account for such size effects due to the lack of a material length scale parameter. Thus, needs exist for the development of size-dependent continuum models which account for these size effects.

In general, size-dependent continuum models can be developed based on size-dependent continuum theories such as couple stress theory [13], nonlocal elasticity theory [14], and strain gradient theory [15].

Based on the modified couple stress theory, proposed by Yang et al. [16], some microstructure-dependent problems had been solved [17–19]. The modified strain gradient theory proposed by Lam et al. [20] results from the classical strain gradient theory [15]. This theory requires three additional material length scale parameters for linear elastic isotropic materials. Recently, this modified theory has been employed by many researchers in order to analyze size-dependent structures. For instance, Kong et al. [21] and Wang et al. [22] investigated static bending and free vibration behaviors of Bernoulli–Euler and Timoshenko homogeneous microbeams, respectively. Static torsion and torsional vibration analysis of clamped–clamped and clamped-free microbars were carried out by Kahrobaiyan et al. [23]. Stability and bending

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responses of microbeams with various boundary conditions were also investigated by Akgöz and Civalek [24,25] on the basis of Bernoulli–Euler beam model. Akgöz and Civalek [26] developed a size-dependent sinusoidal shear deformation beam model in conjunction with modified strain gradient theory. As far as FGM is considered, Ansari et al. [27] investigated the free vibration characteristics of FGM microbeams based on strain gradient Timoshenko beam theory. Akgöz and Civalek [28,29] studied the buckling and vibration of FGM beam or bar based on modified strain gradient theory. Zhang et al. [30] developed a novel size-dependent FGM curved microbeams based on the modified strain gradient theory and nth-order shear deformation theory. Also, Ansari et al. [31] studied bending, buckling and free vibration responses of FGM Timoshenko microbeams based on strain gradient elasticity theory. Sahmani and Ansari [32] presented recently the prediction of buckling behavior of size-dependent FGM third-order shear deformable microbeams including thermal environment effect using modified strain gradient elasticity theory.

To the best of authors' knowledge, however, bending and free vibration of functionally graded piezoelectric beam based on the modified strain gradient theory have not been considered.

In this work, a size-dependent model for bending and free vibration of functionally graded piezoelectric beam is developed using a variational formulation. It is based on the modified strain gradient theory and Timoshenko beam theory. The new model contains three material length scale parameters and can capture the size effect, unlike the classical beam theory. To illustrate the new piezoelectric beam model, the static bending and free vibration problems of a simply supported beam are numerical solved.

2. Formulations of FGPMs

Consider a beam made of functionally graded piezoelectric materials (FGPMs) of length L and thickness h. The beam is subjected to a mechanical load q, and applied voltage V_0 as shown in Fig. 1.

It is considered that the material properties vary continuously across the thickness direction according to the power law distribution. The effective material properties Ξ can be found as:

$$\Xi = \Xi_u V_u(x_3) + \Xi_l V_l(x_3), \tag{1}$$

where (Ξ_u,Ξ_l) represent the material properties at the upper and lower surfaces, respectively, and (V_u,V_l) are the corresponding volume fractions defined as:

$$V_u(x_3) = \left(\frac{x_3}{h} + \frac{1}{2}\right)^{\lambda}, \quad V_l(x_3) = 1 - V_u(x_3), \tag{2}$$

where $\lambda(0 \leqslant \lambda \leqslant \infty)$ denotes the non-negative gradient index.

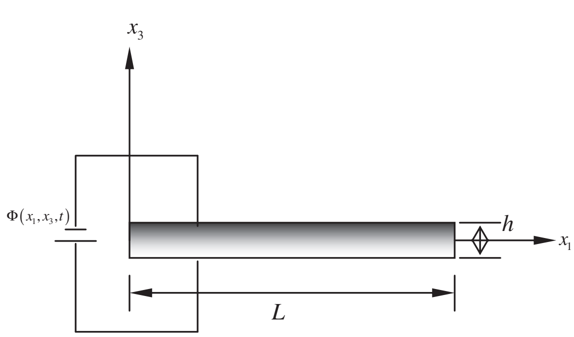

Fig. 1. Schematic configuration of a functionally graded piezoelectric beam.

3. Formulations of modified strain gradient theory

According to the modified strain gradient theory modified by Lam et al. [20], the strain energy of a linear piezoelectric continuum occupying region Ω is written as follows.

$$U = \frac{1}{2}\int_{\Omega}\left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau^{(1)}_{ijk}\eta^{(1)}_{ijk} + m_{ij}\chi_{ij} - D_iE_i\right)dv, \tag{3}$$

where

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{4}$$

$$\gamma_i = \varepsilon_{mm,i}, \tag{5}$$

$$\eta^{(1)}_{ijk} = \frac{1}{3}(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}) - \frac{1}{15}\delta_{ij}(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}) - \frac{1}{15}\left[\delta_{jk}(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}) + \delta_{ki}(\varepsilon_{mm,j} + 2\varepsilon_{mj,m})\right], \tag{6}$$

$$\chi_{ij} = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}), \tag{7}$$

$$\theta_i = \frac{1}{2}(curl(\mathrm{u}))_i, \tag{8}$$

$$E_i = \varphi_{,i}, \tag{9}$$

where u_i, γ_i, θ_i and E_i denote the components of the displacement vector $\mathbf{u}$, the dilatation gradient vector $\boldsymbol{\gamma}$, the infinitesimal rotation vector $\boldsymbol{\theta}$, and the electric field vector $\boldsymbol{E}$. Also, the components of the strain tensor $\boldsymbol{\varepsilon}$, the deviatoric stretch gradient tensor $\boldsymbol{\eta}^{(1)}$, and the symmetric part of the rotation gradient tensor $\boldsymbol{\chi}$ are represented by ε_{ij} $\eta^{(1)}_{ijk}$ and χ_{ij}. The parameters which are obtained by differentiating the strain energy density with respect to kinematics parameters $\boldsymbol{\varepsilon}$, $\boldsymbol{\gamma}$, $\boldsymbol{\eta}^{(1)}$, $\boldsymbol{\chi}$ and $\boldsymbol{E}$ are respectively, symbolized by $\boldsymbol{\sigma}$, $\boldsymbol{p}$, $\boldsymbol{\tau}^{(1)}$, $\boldsymbol{m}$ and $\boldsymbol{D}$. The parameters $\boldsymbol{p}$, $\boldsymbol{\tau}^{(1)}$ and $\boldsymbol{m}$ are usually called the higher-order stresses.

The constitutive equations for the piezoelectric solids may be expressed as follows.

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} + e_{ijk}E_k, \tag{10}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \mu_{ik}E_k, \tag{11}$$

$$p_i = 2\mu l_0^2\gamma_i, \tag{12}$$

$$\tau^{(1)}_{ijk} = 2\mu l_1^2\eta^{(1)}_{ijk}, \tag{13}$$

$$m_{ij} = 2\mu l_2^2\chi_{ij}, \tag{14}$$

where c_{ijkl}, e_{ijk} and μ_{ik} are the elastic, piezoelectric and dielectric coefficients, respectively. μ denotes the shear modules. l_0, l_1 and l_2 are the additional independent material length scale parameters associated with dilatation gradients, deviatoric stretch gradients and symmetry rotation gradients, respectively.

4. Governing equations and boundary conditions

For a piezoelectric beam, the constitutive equation may be expressed as follows:

$$\sigma_{11} = c_{11}\varepsilon_{11} - e_{31}E_3, \tag{15}$$

$$\sigma_{13} = 2c_{55}\varepsilon_{13} - e_{15}E_1, \tag{16}$$

$$D_1 = 2e_{15}\varepsilon_{13} + \mu_{11}E_1, \tag{17}$$

$$D_3 = e_{31}\varepsilon_{11} + \mu_{33}E_3. \tag{18}$$

Based on the Timoshenko beam theory, the displacement field can be expressed as

$$u_1(x_1,x_3,t) = x_3\psi(x_1,t), \tag{19}$$

$$u_3(x_1,x_3,t) = w(x_1,t), \tag{20}$$

where $\psi(x,t)$ is the rotation of the normal to the mid-plane about x directions, respectively.