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Effect of nonlinear elastic foundation on large amplitude free and forced vibration of functionally graded beam

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ABSTRACT

Large amplitude free and forced vibration of FG beam resting on nonlinear elastic foundation containing shearing layer and cubic nonlinearity are investigated. The material properties are assumed to vary continuously according to a simple power law. The theoretical formulations and governing partial deferential equation of motion are derived based on Euler–Bernoulli beam theory and von Karman geometric nonlinearity. Adopting appropriate trial functions for various boundary conditions and employing Galerkin technique and assuming a uniformly distributed harmonic load, single nonlinear ordinary differential equation with quadratic and cubic nonlinearities is obtained. Variational Iteration Method (VIM) is used to derive closed form approximate solutions for both free and forced vibration. Comparison of the acquired results with those of existence literature revealed good agreement with a desired accuracy. The frequency response curves are presented for different coefficients of elastic foundation together with various boundary conditions and the effects of nonlinearities are discussed in detail.

1. Introduction

Significant progress in materials engineering has led to a new class of advanced materials with smooth and continuous variation of thermo-mechanical properties over a changing dimension referred to as functionally graded materials (FGMs). Due to the graded transition in composition across an interface of two materials, FGMs possess remarkable advantages over conventional composite materials such as reduced thermal stresses and smaller stress concentrations in addition to the avoidance of cracking and delamination phenomenon. These characteristics together with the ability of FGMs to be designed and fabricated to satisfy specific purposes have made these revolutionary materials a point of considerable interest in many engineering application fields such as aerospace, biomedical implants, energy, defense, optoelectronics [1–6], etc. Therefore, due to a vast variety of materials and applications, it is of great importance to analyze the response of FGM structures under different, especially mechanical, loadings and study their static and dynamic behaviors. On the other hand beams, as basic mechanical elements, have been used to model many engineering structures in different fields such as civil, marine and aerospace. Although extensive researches have been conducted on dynamical behavior of composite beams based on different beam theories, loadings and materials, flourishing ideas and methods of fabricating FGMs have opened a door to the new area of intensive research on FG beams.

Furthermore, nonlinear vibration analysis to study dynamic behavior at large amplitudes has been an interesting topic among researchers. Due to the nonlinearity in the governing equations and difficulties in deriving analytical solutions, many closed-form approximate solutions have been used. Belendez et al. [7] applied a modified He's homotopy perturbation method to calculate the periodic solutions of nonlinear oscillator with discontinuities. Wang et al. [8] investigated the large deflection problem of a uniform cantilever beam subjected to a terminal concentrated follower force proposing a homotopy perturbation-based method. High accuracy of the method, even in lower ordered perturbations is discussed. A free vibration analysis of nonlinear beam using homotopy and modified Lindstedt–Poincare methods carried out by Ahmadian et al. [9]. Indicating good agreement between semi-analytic solutions with numerical results, HPM is introduced as a powerful tool for analyzing vibration behavior of structures analytically. Zhang [10] adopted the modified Lindstedt–Poincare method to construct the frequency–energy plot of nonlinear vibratory systems. Belendez et al. [11] applied rational harmonic balance method to approximately solve the nonlinear differential equation (Duffing equation) governing the oscillations of conservative autonomous system with one degree of freedom. Peng et al. [12] studied the effects of cubic nonlinear damping on

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vibration isolations using Harmonic balance method. Momeni et al. [13] applied He's energy balance method to Duffing-harmonic oscillators showing that only one iteration leads to high accuracy of the solutions contrary to the conventional methods. Daeichin et al. [14] proposed a novel approach for solving the nonlinear problems based on collocation and energy balance methods. Applying proposed method, frequency–amplitude relationship for nonlinear oscillator with cubic term is obtained and compared with the numerical results. Noor and Mohyud-Din [15] investigated the implementation of He's parameter expansion methods for finding the frequency of nonlinear oscillators. He [16] described a new kind of analytical technique for nonlinear problems called Variational Iteration Method (VIM) and used to give approximate solutions for some well-known nonlinear problems. He and Wu [17] reviewed trends and developments in the use of the VIM and its applications to nonlinear problems arising in various engineering applications. Rafei et al. [18] applied the VIM to nonlinear oscillators with discontinuities and showed that the VIM is an effective and convenient method leading to high accuracy solutions in the first iteration.

Based on the above discussions, there have been several studies on free and forced vibration analysis of FG beams. Ke et al. [19] investigated the nonlinear free vibrations of FG beams based on Euler–Bernoulli beam theory and von Karman geometric nonlinearity employing the direct numerical integration and Runge–Kutta methods. The effects of material property distribution and different end supports on nonlinear dynamic behavior are discussed showing different vibration behavior due to presence of quadratic nonlinearity. Lai et al. [20] derived accurate analytical solutions for large amplitude vibrations of thin FG beams in accordance with the Euler–Bernoulli beam theory and the von Karman type geometric nonlinearity. A generalized Senator–Bapat perturbation technique is employed to solve formulated nonlinear equations. Zarrinzadeh et al. [21] applied a finite element approach to analyze the free vibration characteristics of rotating axially FG tapered beams. The effects of different parameters such as material non-homogeneity, taper ratio and tip mass are studied. Taeprasartsit [22] used a finite element model of large amplitude free vibrations of thin FG beams based on Euler–Bernoulli beam theory and von Karman geometric nonlinearity. Applying a direct iterative method together with the principle of energy conservation, the generated eigenvalue problem is solved. An analysis of nonlinear forced vibrations of clamped FG beams has been carried out by Shooshtari and Rafiee [23] and effects of different parameters on the frequency response have been investigated. Multiple scales method is utilized to solve the second order nonlinear ordinary differential equation derived by applying Galerkin procedure in terms of a time dependent function. Rafiee and Kalhori [24] studied the free and forced Oscillations of FG beams, this time for hinged–hinged and clamped-hinged ends. Again multiple time scales method has been used to obtain natural frequencies for nonlinear problem. Simsek and Kocaturk [25] analyzed the free and forced vibration behavior of the FG Euler–Bernoulli beam subjected to a concentrated moving harmonic load. Lagrange's equations are used to derive the governing equations of motion. The effects of material inhomogeneity, velocity of the moving harmonic load and the excitation frequency on dynamic responses are discussed. Simsek [26], this time, investigated the vibration behavior of a simply-supported FG beam under moving mass by using three different beam theories, i.e. Euler–Bernoulli, Timoshenko and the third order shear deformation beam theories. Again, Lagrange's equations are employed to obtain equations of motion. The effects of various parameters such as the velocity of the moving mass, Coriolis and centripetal effects on dynamic behavior and the stresses of the beam have been discussed in detail. In another study by Simsek [27], nonlinear vibration of a pined–pined FG Timoshenko beam under action of a moving harmonic load is analyzed. Von Karman type geometric nonlinearity has been taken into account and the system of nonlinear equations of motion derived by using Lagrange's equations are solved applying both network-β and direct iteration method. Effects of parameters similar to those mentioned in [26] have been studied. Khalili et al. [28] carried out a similar analysis to [26] but for FG Euler–Bernoulli beam and for both simply supported and clamped–clamped ends using a mixed Rayleigh–Ritz and differential quadrature method. In a recent study, Simsek et al. [29] analyzed the dynamic behavior of an axially functionally graded (AFG) simply supported Euler–Bernoulli beam under action of a moving harmonic load. The same procedure as in [25] has been followed to derive the equations of motion and Newmark method is employed to find dynamic responses.

More recently, there have been few studies on effects of various parameters of elastic foundation on nonlinear behavior of FG beams. Fallah and Aghdam [30] studied large amplitude free vibration and post-buckling of FG beams resting on nonlinear elastic foundation containing shearing layer and cubic nonlinearity subjected to axial force. Using Euler–Bernoulli beam theory and von Karman's strain–displacement relationship and adopting He's variational method, a closed form solution of nonlinear governing equation has been presented. New results concerning the effects of different parameters, particularly coefficients of elastic foundation are presented for simply supported and clamped–clamped boundary conditions. A similar analysis with VIM has been conducted by Yaghoobi and Torabi [31] and nearly same results have been reported. Following the same procedure as in [30], Fallah and Aghdam [32] analyzed thermo-mechanical buckling and nonlinear free vibration of FG beams on nonlinear elastic foundation. Yaghoobi and Torabi [33] applied the same method as in [31] for analysis of geometrically imperfect FG beams.

From the review of the literature, it is clear that most of the researchers, particularly in recent years, are interested in the free and forced nonlinear vibrations of FG beams with more focus on free vibrations. More recently there have been studies on nonlinear free vibrations of FG beams resting on nonlinear elastic foundation; however, to the best of the authors' knowledge, there is no reported work on the forced vibration analysis of FG beams resting on nonlinear elastic foundation.

In this paper, free and forced nonlinear vibrations of FG beams resting on nonlinear elastic foundation containing shearing layer and cubic nonlinearity are investigated. The material properties are assumed to vary continuously according to a simple power law. The theoretical formulations and governing partial deferential equation of motion are derived based on Euler–Bernoulli beam theory and von Karman geometric nonlinearity. Adopting appropriate trial functions for various boundary conditions and employing Galerkin technique and assuming a uniformly distributed harmonic load, single nonlinear ordinary differential equation with quadratic and cubic nonlinearities is obtained. Closed form approximate solutions for both free and forced vibrations are derived using variational iteration method (VIM). In order to validate the acquired results, time response of the forced vibrations is compared with numerical solutions for different amplitudes of excitation and boundary conditions. Also, a comparison study is performed for free vibrations. The frequency response curves are presented for different coefficients of elastic foundation together with various boundary conditions and the effects of nonlinearities are discussed in detail.

2. Governing equation

The physical system analyzed here is a straight FG beam of length l in $\hat{x}$ direction with rectangular cross section of width b and thickness h in $\hat{y}$ and $\hat{z}$ directions, respectively, resting on an nonlinear elastic foundation with cubic nonlinearity and shearing layer and subjected to a harmonic force, as shown in Fig. 1.