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# A quasi-3D theory for vibration and buckling of functionally graded sandwich beams

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ABSTRACT

This paper presents a finite element model for free vibration and buckling analyses of functionally graded (FG) sandwich beams by using a quasi-3D theory in which both shear deformation and thickness stretching effects are included. Sandwich beams with FG skins-homogeneous core and homogeneous skins-FG core are considered. By using the Hamilton's principle, governing equations of motion for coupled axial–shear–flexural–stretching response are derived. The resulting coupling is referred to as fourfold coupled vibration and buckling. Numerical examples are carried out to investigate the thickness stretching effect on natural frequencies and critical buckling loads as well as mode shapes of sandwich beams for various power-law indexes, skin–core–skin thickness ratios and boundary conditions.

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## 1. Introduction

In recent years, there is a rapid increase in the use of functionally graded (FG) sandwich structures in aerospace, marine and civil engineering due to high strength-to-weight ratio. With the wide application of these structures, more accurate theories are required to predict their vibration and buckling response. Amirani et al. [1] used the element free Galerkin method to study free vibration analysis of sandwich beam with FG core. Bui et al. [2] investigated transient responses and natural frequencies of sandwich beams with FG core. Vo et al. [3] studied vibration and buckling of sandwich beams with FG skins – homogeneous core using a refined shear deformation theory. It should be noted that the above mentioned studies ([1–3]) neglected the thickness stretching effect, which becomes very important for thick plates [4]. In order to include shear deformation and thickness stretching effects, the quasi-3D theories, which are based on a higher-order variation through the thickness of the in-plane and transverse displacements, are used. By using these theories, although a lot of work has been done for isotropic and sandwich FG plates ([5–14]), the research on FG sandwich beams is limited. Carrera et al. [15] developed Carrera Unified Formulation (CUF) using various refined beam theories (polynomial, trigonometric, exponential and zig-zag), in which non-classical effects including the stretching effect were automatically taken into account. Recently, he and his co-workers also used CUF to investigate the free vibration of laminated beam [16] and FG layered beams [17]. As far as authors are aware, there is no work available using the quasi-3D theories to study vibration and buckling of FG sandwich beams in a unitary manner. This complicated problem is not well-investigated and there is a need for further studies.

In this paper, which improves the previous research [3] by including the thickness stretching effect, a finite element model for free vibration and buckling analyses of FG sandwich beams by using a quasi-3D theory is presented. Sandwich beams with FG skins-homogeneous core and homogeneous skins-FG core are considered. Governing equations of motion are derived by using the Hamilton's principle. A two-noded $C^1$ beam element with six degree-of-freedom per node is developed. Numerical examples are carried out to investigate the thickness stretching effect on natural frequencies and critical buckling loads as well as mode shapes of sandwich beams for various power-law indexes, skin–core–skin thickness ratios and boundary conditions.

## 2. Theoretical formulation

### 2.1. FG sandwich beams

Consider a FG sandwich beam with length $L$ and rectangular cross-section $b \times h$, with $b$ being the width and $h$ being the height.

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For simplicity, Poisson's ratio $\nu$, is assumed to be constant. Young's modulus $E$ and mass density $\rho$ are expressed by [18]:

$$E(z) = (E_c - E_m)V_c + E_m \tag{1a}$$
$$\rho(z) = (\rho_c - \rho_m)V_c + \rho_m \tag{1b}$$

where subscripts $m$ and $c$ represent the metallic and ceramic constituents, $V_c$ is volume fraction of the ceramic phase of the beam. Two types of FG sandwich beams are considered:

*2.1.1. Type A: sandwich beam with FG skins – homogeneous core*

The bottom and top skin is composed of a FG material, while, the core is ceramic (Fig. 1a). For Type A, $V_c$ is obtained as:

$$\begin{cases} V_c = \left(\frac{z-h_0}{h_1-h_0}\right)^k, & z \in [-h/2, h_1] \quad \text{(bottom skin)} \\ V_c = 1, & z \in [h_1, h_2] \quad \text{(core)} \\ V_c = \left(\frac{z-h_3}{h_2-h_3}\right)^k, & z \in [h_2, h/2] \quad \text{(top skin)} \end{cases} \tag{2}$$

where $k$ is the power-law index.

*2.1.2. Type B: sandwich beam with homogeneous skins – FG core*

The bottom and top skin is metal and ceramic, while, the core is composed of a FG material (Fig. 1b). For Type B, $V_c$ is obtained as:

$$\begin{cases} V_c = 0, & z \in [-h/2, h_1] \quad \text{(bottom skin)} \\ V_c = \left(\frac{z-h_1}{h_2-h_1}\right)^k, & z \in [h_1, h_2] \quad \text{(core)} \\ V_c = 1, & z \in [h_2, h/2] \quad \text{(top skin)} \end{cases} \tag{3}$$

*2.2. Constitutive equations*

The linear constitutive relations are given as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{C}^*_{11} & \bar{C}^*_{13} & 0 \\ \bar{C}^*_{13} & \bar{C}^*_{11} & 0 \\ 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_z \\ \gamma_{xz} \end{Bmatrix} \tag{4}$$

where

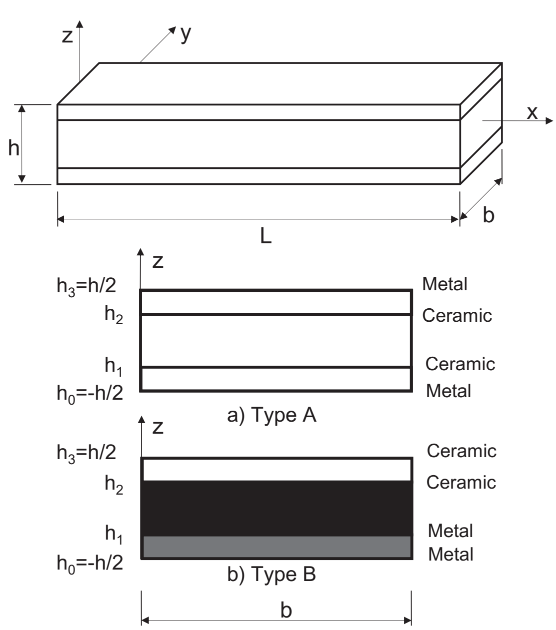


**Fig. 1.** Geometry and coordinate of a FG sandwich beam.

$$\bar{C}^*_{11} = \bar{C}_{11} - \frac{\bar{C}^2_{12}}{\bar{C}_{22}} = \frac{E(z)}{1-\nu^2} \tag{5a}$$
$$\bar{C}^*_{13} = \bar{C}_{13} - \frac{\bar{C}_{12}\bar{C}_{23}}{\bar{C}_{22}} = \frac{E(z)\nu}{1-\nu^2} \tag{5b}$$
$$C_{55} = \frac{E(z)}{2(1+\nu)} \tag{5c}$$

If the thickness stretching effect is omitted ($\epsilon_z = 0$), elastic constants $C_{ij}$ in Eq. (5) are reduced as:

$$\bar{C}^*_{11} = E(z) \tag{6a}$$
$$\bar{C}^*_{13} = 0 \tag{6b}$$
$$C_{55} = \frac{E(z)}{2(1+\nu)} \tag{6c}$$

*2.3. Kinematics*

This paper extends a refined shear deformation theory from previous research [3] by including the thickness stretching effect. The new displacement field is assumed to be [19]:

$$\begin{aligned} U(x,z,t) &= u(x,t) - z\frac{\partial w_b(x,t)}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_s(x,t)}{\partial x} \\ &= u(x,t) - zw_b'(x,t) - f(z)w_s'(x,t) \end{aligned} \tag{7a}$$
$$\begin{aligned} W(x,z,t) &= w_b(x,t) + w_s(x,t) + \left(1 - \frac{4z^2}{h^2}\right)w_z(x,t) \\ &= w_b(x,t) + w_s(x,t) + g(z)w_z(x,t) \end{aligned} \tag{7b}$$

where $u, w_b, w_s$ and $w_z$ are four unknown displacements of mid-plane of the beam. It should be noted that the new component $g(z)w_z(x,t)$ in Eq. (7b) is added to investigate the thickness stretching effect on the vibration and buckling of FG sandwich beams.

The only non-zero strains are:

$$\epsilon_x = \frac{\partial U}{\partial x} = u' - zw_b'' - fw_s'' \tag{8a}$$
$$\epsilon_z = \frac{\partial W}{\partial z} = g'w_z \tag{8b}$$
$$\gamma_{xz} = \frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} = g(w_s' + w_z') \tag{8c}$$

*2.4. Variational formulation*

The variation of the strain energy can be stated as:

$$\begin{aligned} \delta\mathcal{U} &= \int_0^l \int_0^b \left[\int_{-h/2}^{h/2} (\sigma_x\delta\epsilon_x + \sigma_{xz}\delta\gamma_{xz} + \sigma_z g'\delta w_z)dz\right]dydx \\ &= \int_0^l \left[N_x\delta u' - M_x^b\delta w_b'' - M_x^s\delta w_s'' + Q_{xz}(\delta w_s' + \delta w_z') + R_z\delta w_z\right]dx \end{aligned} \tag{9}$$

where $N_x, M_x^b, M_x^s, Q_{xz}$ and $R_z$ are the stress resultants, defined as:

$$N_x = \int_{-h/2}^{h/2} \sigma_x b dz \tag{10a}$$
$$M_x^b = \int_{-h/2}^{h/2} \sigma_x z b dz \tag{10b}$$
$$M_x^s = \int_{-h/2}^{h/2} \sigma_x f b dz \tag{10c}$$
$$Q_{xz} = \int_{-h/2}^{h/2} \sigma_{xz} g b dz \tag{10d}$$
$$R_z = \int_{-h/2}^{h/2} \sigma_z g' b dz \tag{10e}$$

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