Composite Structures 119 (2015) 134–149

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Layerwise mixed models for analysis of multilayered piezoelectric composite plates using least-squares formulation

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ARTICLE INFO

Article history:
Available online 4 September 2014

Keywords:
Layerwise mixed formulation
Least-squares formulation
Finite element model
Electromechanical analysis
Piezoelectric composite plates

ABSTRACT

This work provides an assessment of layerwise mixed models using least-squares formulation for the coupled electromechanical static analysis of multilayered plates. In agreement with three-dimensional (3D) exact solutions, due to compatibility and equilibrium conditions at the layers interfaces, certain mechanical and electrical variables must fulfill interlaminar C^0 continuity, namely: displacements, in-plane strains, transverse stresses, electric potential, in-plane electric field components and transverse electric displacement (if no potential is imposed between layers). Hence, two layerwise mixed least-squares models are here investigated, with two different sets of chosen independent variables: Model A, developed earlier, fulfills *a priori* the interlaminar C^0 continuity of all those aforementioned variables, taken as independent variables; Model B, here newly developed, rather reduces the number of independent variables, but also fulfills *a priori* the interlaminar C^0 continuity of displacements, transverse stresses, electric potential and transverse electric displacement, taken as independent variables. The predictive capabilities of both models are assessed by comparison with 3D exact solutions, considering multilayered piezoelectric composite plates of different aspect ratios, under an applied transverse load or surface potential. It is shown that both models are able to predict an accurate quasi-3D description of the static electromechanical analysis of multilayered plates for all aspect ratios.

1. Introduction

With the recent availability of stable and high performance sensors and actuators, numerous applications of smart structures technology are currently evolving to actively control structural vibration, damping, noise, aeroelastic stability, shape change and stress distribution. A smart structure incorporates smart material sensors and actuators that enable it to monitor a change in its environment, whether external (such as loads or shape change) or internal (such as damage or failure), and adapt to it by modifying the system characteristics (such as stiffness or damping) or the system response (such as strain or shape), in a controlled manner. Among smart materials, piezoelectric sensors and actuators are being used extensively due to their excellent electromechanical properties, including fast response, large operating bandwidth and low power consumption, as well as their easy fabrication, low cost and design flexibility, such that they can be easily integrated in multilayered composite structures. In fact, smart composite structures offer the possibility to combine the lightweight, superior mechanical and thermal properties of composite materials with actuation, sensing and control. Due to their self-monitoring and self-adaptive capability, smart composite structures technology has an enormous potential to impact high performance engineering applications.

An accurate electromechanical modeling of these smart composite structures, which are inherently anisotropic multilayered structures, requires an appropriate description of both mechanical and electrical variables, particularly in the thickness direction. As demonstrated by 3D exact piezoelasticity solutions developed by Heyliger [1,2] and also by Heyliger and Saravanos [3] for static and free vibration analysis of simply supported multilayered, orthotropic, piezoelectric composite plates, these structures may exhibit complicating effects due to different mechanical and electric properties in the thickness direction. Actually, early on, the pioneer work of Pagano [4] on the 3D exact elasticity solutions for static analysis of simply supported multilayered, orthotropic, composite plates had already shown the effects introduced by both in-plane anisotropy, as high transverse deformability, and transverse anisotropy, as zig-zag effects and interlaminar continuity. In fact, due to compatibility and equilibrium conditions at the

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http://dx.doi.org/10.1016/j.compstruct.2014.08.031

layers interfaces, certain mechanical and electrical variables must fulfill interlaminar C^0 continuity, namely: displacements, in-plane strains, transverse stresses, electric potential, in-plane electric field components and transverse electric displacement (if no potential is imposed between layers). The fulfillment of these C_z^0-Requirements is crucial for an accurate electromechanical modeling of multilayered structures. Comprehensive assessments on the modeling and analysis of multilayered structures are available in excellent review papers by Carrera [5–7] as well as in the book of Reddy [8], and particularly considering piezoelectric structures, in well-known selected papers [9–13]. In overview, the finite element models mostly differ in equivalent single layer (ESL) or layerwise (LW) variable descriptions as well as in the chosen unknown variables, consistent with classical (i.e. displacement-based) or mixed formulations. In fact, although ESL models do provide a reasonably good global analysis of multilayered structures, LW models are much better suited to reach an accurate analysis, particularly in the case of very thick structures. The models are commonly derived from a variational principle consistent with the chosen formulation: classical models are based on the principle of virtual displacements; and the alternative mixed models are usually based on the Reissner mixed variational principle. The aforementioned 3D exact solutions have been extremely useful in assessing the accuracy of numerous plate theories and related finite element models. As a matter of fact, given the limited number of test cases whose 3D exact solutions have been published, the authors recently provided additional test cases to serve as benchmark 3D exact solutions [14] for the static analysis of multilayered piezoelectric composite plates by successfully implementing the method formerly introduced by Heyliger.

Some relevant contributions to the modeling of multilayered piezoelectric composite plates are worth mentioning. Heyliger et al. [15] developed a LW classical model for static analysis using as primary variables the displacements and electric potential, assuming a simple linear z-expansion through the layer thickness, and later Saravanos et al. [16] provided its extension to dynamic analysis. Semedo Garção et al. [17] also developed a LW classical model for static analysis using the same primary variables, but allowing a higher-order z-expansion through the layer thickness, so as to improve the accuracy of transverse stresses and transverse electric displacement. However, as much emphasized in the works of Carrera [5–7], initially on purely mechanical analysis of multilayered composite structures, and later extended to the coupled electromechanical analysis of piezoelectric plates [11–13], the alternative LW mixed models are able to ensure *a priori* the fulfillment of the interlaminar C^0 continuity of all chosen variables used in the mixed formulation, which may include displacements, transverse stresses, electric potential and transverse electric displacement. Therefore, LW mixed models are capable to furnish a more accurate description of all chosen mechanical and electrical variables in agreement with the C_z^0-Requirements. In fact, Lage et al. [18] developed a LW mixed model for static analysis using precisely as primary variables the displacements, transverse stresses, electric potential and transverse electric displacement, assuming a quadratic z-expansion through the layer thickness. More recently, Carrera et al. [19–22] provided some distinct LW mixed models for both static and dynamic analysis, using different sets of chosen primary variables, keeping the displacements and electric potential, along with the transverse stresses and/or the transverse electric displacement, and allowing up to 4th-order z-expansion through the layer thickness.

As previously mentioned, finite element models are commonly derived from a variational principle, as the principle of virtual displacements or the Reissner mixed variational principle (i.e. weak formulation). These weak form models are sometimes called Ritz models. Alternatively, finite element models can be derived from a weighted residual formulation (i.e. strong formulation). In this case, different types of models arise in agreement with the chosen weight functions, including collocation models, Galerkin models, and in particular least-squares models. All the aforementioned LW classical and mixed models are actually weak form models. The benefit of a least-squares model combined with a mixed formulation is that it is able to by-pass inf–sup conditions [23], and leads to a symmetric positive definite algebraic problem, as opposed to a mixed weak form model. An added benefit of a least-squares model is that it also appears to be insensitive to shear-locking. The authors already developed in earlier works [24,25] a LW mixed least-squares model for both static and free vibration analysis of multilayered composite plates, which fulfills *a priori* the interlaminar C^0 continuity of displacements, in-plane strains and transverse stresses, taken as independent variables. Note that to be exact the term primary (and secondary) variable applies only to a weak formulation. Such earlier model was then extended to multilayered piezoelectric composite plates by the authors [26], adding the electric potential, in-plane electric field components and transverse electric displacement, as independent variables as well. This LW mixed least-squares model which completely fulfills *a priori* the C_z^0-Requirements is henceforth designated Model A, whose results are here included for comparison purposes. Presently, the authors developed a new LW mixed least-squares model that rather reduces the number of independent variables, by discarding the in-plane strains and in-plane electric field components. This new model presented here is henceforth designated Model B, which also fulfills *a priori* the interlaminar C^0 continuity of displacements, transverse stresses, electric potential and transverse electric displacement, taken as independent variables.

This work aims to assess the predictive capabilities of both Model A and Model B by comparison with 3D exact solutions, considering multilayered piezoelectric composite plates of different aspect ratios, under an applied transverse load or surface potential. The idea is to check whether the lower computational effort of Model B compromises in any way its accuracy compared to Model A. For assessment purposes, the order of the two-dimensional approximations in-plane and the order of the z-expansion through the layer thickness, as well as the number of elements and number of layers, are considered free parameters for both models. In fact, in order to properly minimize the least-squares functional, high-order basis functions are typically adopted, both in-plane and through the layer thickness, along with full integration. Hence, the number of elements and number of layers are usually kept to a reasonable minimum and refinement is largely achieved through higher-order basis functions.

2. Layerwise mixed least-squares models

Consider a multilayered plate of total thickness h and rectangular planar geometry Ω made of N generally orthotropic layers, such as piezoelectric or composite layers, as shown in Fig. 1. A Cartesian coordinate system (x,y,z) is used, with the z-axis taken positive upward from the midplane along the thickness direction. Also, the superscript k is assigned to the kth layer specifics.

Two layerwise mixed least-squares models are here investigated, with two different sets of chosen independent variables, as summarized in Table 1. Accordingly, Model A involves a total of 13 independent variables for each layer, whereas Model B reduces this number to 8 to lower the computational effort.

2.1. Governing equations for each layer

In accordance with linear piezoelasticity the complete set of equations for the linear static analysis of each layer, which is treated as a piezoelectric layer in general, consists of the equilibrium equations, charge equation of electrostatics, constitutive equations, strain–displacement equations and field–potential equations [1–3], as follows: