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Nonuniform biaxial buckling of orthotropic nanoplates embedded in an elastic medium based on nonlocal Mindlin plate theory

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ABSTRACT

In this article, non-uniform biaxial buckling analysis of orthotropic single-layered graphene sheet embedded in a Pasternak elastic medium is investigated using the nonlocal Mindlin plate theory. All edges of the graphene sheet are subjected to linearly varying normal stresses. The nanoplate equilibrium equations are derived in terms of generalized displacements based on first-order shear deformation theory (FSDT) of orthotropic nanoplates using the nonlocal differential constitutive relations of Eringen. Differential quadrature method (DQM) has been used to solve the governing equations for various boundary conditions. The accuracy of the present results is validated by comparing the solutions with those reported by the available literatures. Finally, influences of small scale effect, aspect ratio, polymer matrix properties, type of planar loading, mode numbers and boundary conditions are discussed in details.

1. Introduction

Graphene, first discovered by Geim and Novoselov [1] in 2004, is a monolayer (with a thickness of only ~0.34 nm) of sp2 hybridized carbon atoms (covalently bonded to three other atoms) arranged in a honeycomb lattice with a unique series of unprecedented structural, mechanical and electrical properties [2]. The superior properties of these structures have led to its applications in many fields such as nano sensors, electrical batteries, superfast microelectronics, micro electromechanical systems (MEMS), nano-electromechanical systems (NEMS), biomedical, bioelectrical, reinforcement role at composites, etc. [3–9]. The most commonly employed methods for graphene manufacturing are micromechanical and chemical exfoliation of graphite, reduction of graphite oxide, epitaxial growth on SiC, and chemical vapor deposition (CVD) on transition metals [10]. In order to study the mechanical behavior of nanostructures, it has been reported that the small scale effect must play an important role in the nanoscale structures, but this small scale effect has been ignored when classical local continuum theory was adopted [11]. Really, we cannot neglect the van der Waals interaction between the atoms and its inner distance in contrast with the main structure [12]. On the other hand, performing the experiment at nanoscale is very difficult and expensive; also the atomistic simulation such as molecular dynamics (MD) is highly computationally expensive and cannot be applied for more number of atoms at surface. So, using some other methods is vital. In recent years, various size-dependent continuum theories such as couple stress theory [13], strain gradient elasticity theory [14–16], modified couple stress theory [17–21] and nonlocal elasticity theory [22–25] are proposed. These theories are comprised of information about the interatomic forces and internal lengths that is introduced as small scale effect in nonlocal elasticity theory [25]. Chen [26] employing lattice dynamics and MD showed that among the size-dependent continuum theories (micromorphic theory, microstructure theory, micropolar theory, Cosserat theory, nonlocal theory and couple stress theory), the nonlocal elasticity theory of Eringen is the most reasonable from the physical and atomic points of view. Also, Sun et al. [27] found that there exists an inconsistency between atomistic simulation and the strain gradient elasticity solution for the bending of nano-scale structures. Peddieson et al. [28] first used the nonlocal elasticity theory to develop a nonlocal Bernoulli/Euler beam model. After that, the nonlocal elasticity theory has been widely used due to its simplicity, high reliability and close agreement with MD simulations for mechanical analysis of carbon nanotubes and graphene sheets [26,27,29]. However, on contrary to the huge studies presented for mechanical analysis of one-dimensional nanostructures such as nanobeams, nanorods and CNTs, only some works are presented on two-dimensional ones such as graphene sheets. So, our understanding of the mechanical behaviors of

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graphene sheets such as buckling is essential for their engineering design and manufacture. Pradhan [30] employed higher-order shear deformation theory using the nonlocal differential constitutive relations of Eringen in order to study buckling behavior of isotropic single-layered graphene sheet and obtained an analytical solution for critical buckling load of these nanoplates. Murmu and Pradhan [31] carried out biaxial buckling study of orthotropic graphene sheets based on nonlocal Kirchhoff model and obtained explicit expression for modified buckling load. They [32] also implemented nonlocal elasticity theory to investigate the buckling behavior of single-layered graphene sheet (SLGS) embedded in an elastic medium. Their results show that the buckling loads of SLGS are strongly dependent on the small scale coefficients and the stiffness of the surrounding elastic medium. Narendar [33] presented the buckling analysis of isotropic graphene sheets using the two variable refined plate theory and nonlocal small scale effects. He concluded that the present theory, which does not require shear correction factor, is not only simple but also comparable to the first-order and higher order shear deformable theory. Samaei et al. [34] investigated the effect of length scale on buckling behavior of an isotropic single-layer graphene sheet embedded in a Pasternak elastic medium using a nonlocal Mindlin plate theory and extracted explicit solution for the buckling loads of graphene sheet. Farajpour et al. [35] studied uniaxial buckling response of orthotropic nanoscale plates under linearly varying in-plane load via nonlocal Kirchhoff theory. They found that for the case of pure in-plane bending, the nonlocal effects are relatively more than other cases. Using Levy's method, Pradhan [36] investigated buckling behavior of biaxially compressed graphene sheets based on nonlocal elasticity theory. He found that nonlocal parameter and boundary conditions significantly influence the critical buckling loads of the small size graphene sheets. Ansari and Shamani [37] studied the biaxial buckling behavior of single-layered graphene sheets based on nonlocal plate models and molecular dynamic simulations. They extracted the appropriate values of nonlocal parameter relevant to each type of nonlocal elastic plate model and chirality. They also showed that the present nonlocal plate models with their proposed proper values of nonlocal parameter have an excellent capability to predict the biaxial buckling response of SLGSs. Analooei et al. [38] used nonlocal continuum mechanics and spline finite strip method due to elastic buckling and vibration analysis of orthotropic nanoplates. Their results revealed that small scale effect plays considerable role in the analysis of small sizes plates. Murmu et al. [39] reported an analytical study on the buckling of double-nanoplate-system (DNPS) subjected to biaxial compression using nonlocal elasticity theory. Sarrami-Foroushani and Azhari [40] using the nonlocal classical plate theory and finite strip method studied vibration and buckling of single and multi-layered graphene sheets. They observed that in the nanoscale structures the critical buckling load and natural frequency are highly dependent on nonlocal parameter.

As far as knowledge of authors is concerned, there is no literature considering the non-uniform biaxial buckling analysis of orthotropic nanoplate embedded in a Pasternak elastic medium based on the nonlocal Mindlin plate theory. Thus, this study is presented considering the non-uniform biaxial buckling of embedded graphene sheet under various distribution of linearly planar load along the edges. Governing equations are derived based on Mindlin theory with considering orthotropic property and nonlocal theory of Eringen in order to consider the size effects. Both Winkler-type and Pasternak-type foundation models are employed to simulate the interaction between the graphene sheet and the surrounding elastic medium. The created eigenvalue problem is solved using the differential quadrature method for simply-supported boundary condition, clamped boundary condition and combination of them. To verify the accuracy of the present consequences, simplified results are compared with those of available references and molecular dynamics results. Excellent agreement between the results is observed fortunately. Finally, influences of many parameters such as small scale effect, aspect ratio, polymer matrix properties, distribution of planar loading, mode numbers and boundary conditions are discussed in details.

2. Formulation

The single-layered graphene sheet is simulated as a rectangular nano-plate and the elastic medium is modeled using an elastic foundation, both Winkler-type and Pasternak-type elastic foundation. Fig. 1 shows the discrete model and continuum model used in this study for a single-layer graphene sheet resting on two-parameter foundation with length l_x, width l_y and thickness h. As seen in Fig. 1, a Cartesian coordinate system is placed at one corner of the graphene sheet with the x,y and z axes along the length, width and thickness, respectively. Also, the linear variations of in-plane compressive loadings along the x and y axes denoted in Fig. 2 by $P_y(x)$ and $P_x(y)$, respectively, are defined by:

$$P_x(y) = -P_1\left(1 - k_1\frac{y}{l_y}\right), \quad P_y(x) = -P_2\left(1 - k_2\frac{x}{l_x}\right) = -k_0P_1\left(1 - k_2\frac{x}{l_x}\right) \tag{1}$$

where P_1 is normal stress along the x direction at origin point and k_0, k_1, k_2 are optional parameters defined to express the loading distribution. According to the first-order shear deformation theory (FSDT), the following displacement field can be expressed as:

$$\begin{cases} u(x,y,z,t) = u_0(x,y,t) + z\varphi_x(x,y,t) \\ v(x,y,z,t) = v_0(x,y,t) + z\varphi_y(x,y,t) \\ w(x,y,z,t) = w_0(x,y,t) \end{cases} \tag{2}$$

where u, v and w are the displacement components of point (x,y) along x, y and z directions, respectively at time t. Also, u_0, v_0 and w_0 are the displacement functions of the middle surface of the graphene sheet. Moreover, φ_x and φ_y are the local rotations for x and y directions, respectively. The general strain relations are expressed as:

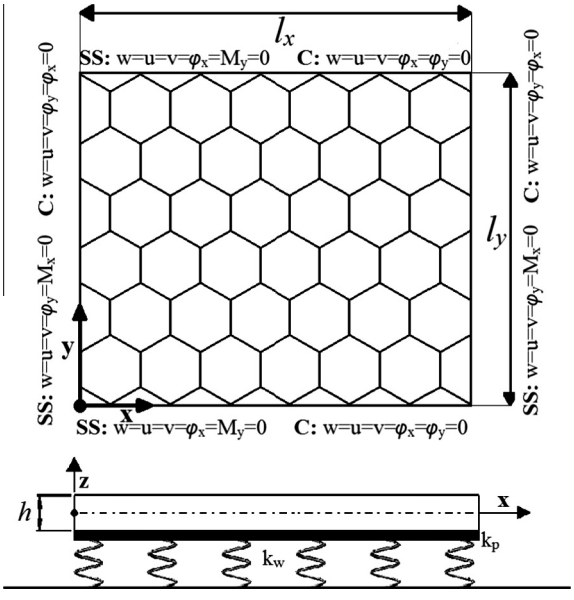

Fig. 1. Graphene sheet in a Pasternak medium under biaxial buckling load.