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Nonlinear free vibration of magneto-electro-elastic rectangular plates

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ABSTRACT

Nonlinear free vibration of symmetric magneto-electro-elastic laminated rectangular plates with simply supported boundary condition is studied for the first time. The first order shear deformation theory considering the von Karman's nonlinear strains is used to obtain the equations of motion, whereas Maxwell equations for electrostatics and magnetostatics are used to model the electric and magnetic behavior. Closed circuit electro-magnetic boundary condition at top and bottom surfaces of the plate is considered. Then, the nonlinear partial differential equations of motion are transformed into five coupled nonlinear ordinary differential equations by using the Galerkin method. Afterward, the obtained coupled ordinary differential equations are reduced to a single nonlinear differential equation with quadratic and cubic nonlinear terms. A perturbation method is used to solve the equation of motion analytically and a closed-form solution is obtained for the nonlinear frequency ratio. The results for natural frequency and nonlinear frequency ratio are compared with the available results for isotropic, laminated and piezo-laminated, and laminated magneto-electro-elastic plates and good agreement is found between the results of present study with the results of previously published papers. Several numerical examples are carried out to show the effects of different parameters on the nonlinear behavior of these hybrid plates.

1. Introduction

Magneto-electro-elastic (MEE) composite materials are a new class of smart materials which exhibit a coupling between mechanical, electric and magnetic fields and because of the ability of converting energy among these three energy forms, these laminates have direct application in sensors and actuators, control of vibrations in structures, etc.

Various analytical or numerical studies have been carried out for these multiphase and multifunctional materials which include studies on the static deformation [1,2], free vibration [3–9] and on the linear dynamic response [10–15]. Milazzo [16] presented a family of 2D refined equivalent single layer models for multilayered and functionally graded smart magneto-electro-elastic plates. The same author [17] introduced layer-wise and equivalent single layer model to study the multi-layered laminated MEE plates. Li et al. [18] investigated the buckling and free vibration of magneto-electroelastic nanoplate resting on Pasternak foundation and studied the effects of the electric and magnetic potentials, and spring and shear coefficients of the Pasternak foundation on the buckling load and vibration frequency.

In the nonlinear field, few studies on the nonlinear behavior of MEE plates are available which deal with static motion [19–22] or the nonlinearity is due to constitutive equations of MEE material [23,24]. Xue et al. [19] studied the large deflection of a rectangular MEE plate for the first time. They derived the nonlinear partial differential equation (PDE) of motion for the rectangular MEE thin plate based on the von Karman plate theory of large deflection and by employing the Bubnov–Galerkin method, transformed these PDEs to a third order algebraic equation. Then by solving this algebraic equation, they found an analytical relation for maximum deflection of the plate. Later, Sladek et al. [20] used a meshless local Petrov–Galerkin (MLPG) method to study the large deflection of MEE thick plates under a static and time-harmonic mechanical load. Milazzo [21] derived a model for the large deflection analysis of magneto-electro-elastic laminated plates based on the first order shear deformation theory and the von Karman stress function. Alaimo et al. [22] presented an original finite element formulation for the analysis of large deflections in magneto-electro-elastic multilayered plates. They used first order shear deformation theory with von Karman strains and quasi-static behavior for the electric and magnetic fields to obtain their model. More recently, Kattimani and Ray [25] studied the active constrained layer damping (ACLD) of large amplitude vibrations of smart MEE doubly curved shells by using a three-dimensional finite element model

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using the von Karman type nonlinear strain displacement relations for incorporating the geometric nonlinearity. To the authors knowledge there is not any study dealing with the nonlinear vibration of MEE smart plates. So, this study is done to fill this gap in analyzing of MEE plates.

In this paper, the nonlinear free vibration of a symmetrically stacked laminated MEE rectangular plate with simply supported boundary condition is studied for the first time based on the first order shear deformation theory along with the von Karman's nonlinear strains, whereas the Maxwell equations for electrostatics and magnetostatics are used to model the electric and magnetic behavior. Closed circuit electro-magnetic boundary condition at top and bottom surfaces of the plate is considered. Moreover, the widely employed assumption of zero in-plane components of the electric and magnetic fields are considered in the present study, namely E_x, E_y, H_x, and H_y are neglected and only the transverse electric field E_z and magnetic field H_z are considered. A perturbation method is used to solve the equation of motion analytically. After the validation of present study, the effects of several parameters on the nonlinear behavior of MEE plates are investigated.

2. Modeling of the problem

Consider a rectangular transversely isotropic MEE thin plate as shown in Fig. 1 in which a, b, and h are length, width and thickness of the plate, respectively. For a MEE material, the constitutive relations can be written as [1]:

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} - \mathbf{e}\mathbf{E} - \mathbf{q}\mathbf{H} \tag{1}$$

$$\mathbf{D} = \mathbf{e}^T\boldsymbol{\varepsilon} + \boldsymbol{\eta}\mathbf{E} + \mathbf{d}\mathbf{H} \tag{2}$$

$$\mathbf{B} = \mathbf{q}^T\boldsymbol{\varepsilon} + \mathbf{d}\mathbf{E} + \boldsymbol{\mu}\mathbf{H} \tag{3}$$

in which for an orthotropic MEE solid, the coefficients are given by [1]:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 & 0 \\ 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} 0 & 0 & q_{31} \\ 0 & 0 & q_{32} \\ 0 & q_{24} & 0 \\ q_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\eta} = \begin{bmatrix} \eta_{11} & 0 & 0 \\ 0 & \eta_{22} & 0 \\ 0 & 0 & \eta_{33} \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \tag{4}$$

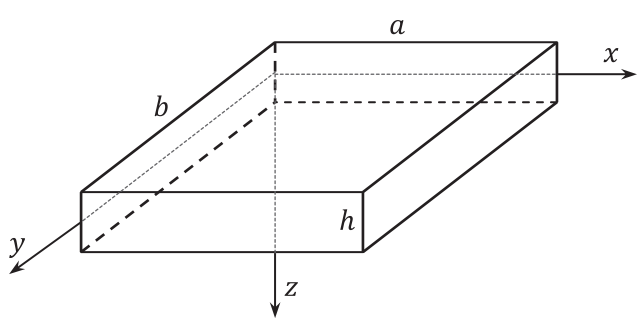

Fig. 1. A thin rectangular MEE plate.

where $\boldsymbol{\sigma} = \{\sigma_{xx}\ \sigma_{yy}\ \sigma_{yz}\ \sigma_{xz}\ \sigma_{xy}\}^T$ and $\varepsilon = \{\varepsilon_{xx}\ \varepsilon_{yy}\ \gamma_{yz}\ \gamma_{xz}\ \gamma_{xy}\}^T$ are stress and strain vectors, respectively; $\mathbf{D} = \{D_xD_yD_z\}^T$ and $\mathbf{B} = \{B_xB_yB_z\}^T$ are the electric displacement and magnetic flux vectors, respectively; $\mathbf{E} = \{E_xE_yE_z\}^T$ and $\mathbf{H} = \{H_xH_yH_z\}^T$ are electric field and magnetic field vectors, respectively; $\mathbf{C}$, $\boldsymbol{\eta}$ and $\boldsymbol{\mu}$ are the elastic, dielectric and magnetic permeability coefficient matrices, respectively; and $\mathbf{e}$, $\mathbf{q}$ and $\mathbf{d}$ are the piezoelectric, piezomagnetic and magnetoelectric coefficient matrices, respectively.

Equations of motion of rectangular plates, based on the first order shear deformation theory are [26]:

$$N_{x,x} + N_{xy,y} = I_0 u_{0,tt} + I_1\theta_{x,tt} \tag{5}$$

$$N_{xy,x} + N_{y,y} = I_0 v_{0,tt} + I_1\theta_{y,tt} \tag{6}$$

$$Q_{x,x} + Q_{y,y} + \mathcal{N}(w_0) + q = I_0 w_{0,tt} \tag{7}$$

$$M_{x,x} + M_{xy,y} - Q_x = I_2\theta_{x,tt} + I_1 u_{0,tt} \tag{8}$$

$$M_{xy,x} + M_{y,y} - Q_y = I_2\theta_{y,tt} + I_1 v_{0,tt} \tag{9}$$

where subscript ',' denotes partial differentiation with respect to the following parameter (or parameters). u_0, v_0, and w_0 are the displacements of a material point on the mid-surface along $x-$, $y-$, and $z-$axes, respectively. θ_x and θ_y are the rotations of a transverse normal about the y- and x-axes, respectively. N_x, N_y, and N_{xy} are the in-plane force resultants, Q_x and Q_y are the transverse force resultants, M_x, M_y, and M_{xy} are the moments resultants and I_0, I_1, and I_2 are the mass moments of inertia. q is the applied transverse load which is zero in the free vibration. $\mathcal{N}(w_0)$ is:

$$\mathcal{N}(w_0) = \left(N_x w_{0,x} + N_{xy} w_{0,y}\right)_{,x} + \left(N_{xy} w_{0,x} + N_y w_{0,y}\right)_{,y} \tag{10}$$

and the other unknown parameters are obtained by:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z dz,$$
$$\begin{Bmatrix} I_0 \\ I_1 \\ I_2 \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} 1 \\ z \\ z^2 \end{Bmatrix} \rho_0 dz, \quad \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = K\int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{11}$$

where K is the shear correction factor.

Assuming the density of plate material (ρ_0) as an even function of thickness (z) and neglecting in-plane inertia effects ($u_{0,tt}$ and $v_{0,tt}$) [27] and rotary inertial effects ($\theta_{x,tt}$ and $\theta_{y,tt}$), Eqs. (5)–(9) reduce to:

$$N_{x,x} + N_{xy,y} = 0 \tag{12}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{13}$$

$$Q_{x,x} + Q_{y,y} + \left(N_x w_{0,x} + N_{xy} w_{0,y}\right)_{,x} + \left(N_{xy} w_{0,x} + N_y w_{0,y}\right)_{,y} = I_0 w_{0,tt} \tag{14}$$

$$M_{x,x} + M_{xy,y} - Q_x = 0 \tag{15}$$

$$M_{xy,x} + M_{y,y} - Q_y = 0 \tag{16}$$

Substitution of Eqs. (1)–(4) along with von Karman's nonlinear strains [26] into Eq. (11) gives the following force and moment resultants for a symmetric MEE plate:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} u_{0,x} + \frac{1}{2}w_{0,x}^2 \\ v_{0,y} + \frac{1}{2}w_{0,y}^2 \\ u_{0,y} + v_{0,x} + w_{0,x}w_{0,y} \end{Bmatrix}$$
$$- \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -\phi_{,z} \end{Bmatrix} dz - \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} 0 & 0 & q_{31} \\ 0 & 0 & q_{32} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -\psi_{,z} \end{Bmatrix} dz \tag{17}$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = K \begin{bmatrix} A_{44} & 0 \\ 0 & A_{55} \end{bmatrix} \begin{Bmatrix} w_{0,y} + \theta_y \\ w_{0,x} + \theta_x \end{Bmatrix} \tag{18}$$