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A semi analytical approach for large amplitude free vibration and buckling of nonlocal FG beams resting on elastic foundation

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ABSTRACT

In this paper, an attempt is made to obtain a closed form solution for both natural frequency and buckling load of nonlocal FG beams resting on nonlinear elastic foundation. Implementing Eringen's nonlocal elasticity theory, the effect of nonlocality is introduced into the Euler–Bernoulli beam theory to obtain the nonlinear governing partial differential equation. Application of the Galerkin technique to the governing equation leads to a nonlinear ODE in the time-domain. Finally, natural frequency of the FG nano beam is obtained using He's variational method. It is shown that considering the nonlocal effects decreases the buckling load as well as natural frequency. Results also reveal that effects of nonlocal parameters on fully clamped beams are more than other types of boundary conditions. Moreover, it is shown that the effect of nonlocality decreases by increasing length of the beam.

1. Introduction

Half a century passed from Richard Feynman lecture in American Physical Society about the possibility of maneuvering things atom by atom [1], development in modern technologies leads to create many new materials at nano scales. Nanotechnology is the manipulation of matter on an atomic, molecular, and supramolecular scale. Nanotechnology has been successful in creating many new applied materials and devices in electronics, medicine and energy production. The distinguished properties of materials at nano scale are resulted from their very small dimensions. Both experimental and atomistic simulation approves that size effect has a significant role in static and dynamic behavior of nano structures. Mainly, size effects arise from two sources; existence of quantum effects and high surface to volume ratio [2].

It is well known that size effects are not taken into account by the classical continuum mechanics and therefore, cannot be used directly to model material behavior at this scale. On the other hand, however, atomistic analyses or experimental tests demand higher computational or laboratory costs. Therefore, in order to include the size effects in continuum mechanics, different theories are proposed to include additional length parameters, such as modified couple stress theory [3], strain gradient theory [4], nonlocal elasticity theory [5] and surface elasticity [6]. Among these theories, Eringen's nonlocal elasticity theory [5] is shown to be capable of studying different behavior of nano structures. Subsequently, this theory was implemented by various researchers in order to investigate the mechanical behavior of different structural elements. Peddison et al. [7] were the first researchers to propose nonlocal elasticity theory to nano structures. Afterward, the theory received more attention among the nanotechnology community and the application of this theory generalized in different mechanical analyses.

The simplicity of application of Eringen's nonlocal elasticity theory in different nano structures resulted in rapid extension of this theory. Wang and Liew [8] used the model to perform a static analysis of micro and nano structures. Reddy [9] combined the nonlocal theory with different types of beam theories to analyze bending, buckling and vibration of nanobeams. Eringen's nonlocal theory was also of interest in the modeling of carbon nanotubes. Pradhan and Reddy [10] applied DTM to analyze buckling of single walled carbon nanotubes which is resting on Winkler elastic foundation. Zhang et al. [11,12] studied the effect of small length scale on the elastic buckling of multi-walled carbon nanotubes under axial compression and radial pressure, respectively. Murmu and Pradhan [13] combined Timoshenko beam theory with principles of nonlocal elasticity to study the buckling of single-walled carbon nanotube embedded in an elastic medium. Phadikar and Pradhan [14] also used variational formulation in finite element analysis of nonlocal elastic nanobeams and nanoplates. Murmu and Adhikari [15] analyzed the transverse vibration of double-nanobeam-systems. More recently, Thai and Vo [16] presented a

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nonlocal sinusoidal shear deformation beam theory for bending, buckling and vibration of nanobeams.

In the past decade, functionally graded materials (FGMs) were also of interest. This new type of materials with continuous variation of the material properties have some advantages over conventional laminates such as smaller thermal stress and stress concentration which reduce possibility of delamination and cracking. The application of functionally graded materials are in nano/micro electromechanical systems such as atomic force microscopes [17] and electrically actuated MEMS devices [18,19]. The dimensions of these structural devices usually do not exceed micron scale; therefore, a size dependent analysis seems to be necessary in investigation of FG materials. Up to now, different theoretical and experimental investigations have been performed in order to analyze mechanical behavior of FG structures, see [20,21]. Moreover, different types of nano scale FG materials are produced by using various fabrication processes Kian Kerman et al. [22] applied co-sputtering to fabricate a compositionally graded electrolyte at nano scale which is employed in low-temperature solid oxide fuel cell and is mechanically robust and chemically stable for this application. Kim et al. [23], also, discussed the fabrication of nano-micro porous Titanium surface by anodizing which exhibited enhanced performance. Wang et al. [24] applied centrifugal method to fabricate functionally graded nanocomposites. Stepwise functionally graded synthetic nanocomposites were also fabricated by combined powder stacking and compression molding techniques [25].

Recently, a few researchers made attempts to consider nonlocal effects in the analysis of structures made of functionally graded materials. Eltaher et al. [26,27] obtained the natural frequencies and investigated static and stability behavior of functionally graded nanobeams by using finite element formulation. Şimşek [28] implemented Galerkin technique to investigate the behavior of axially functionally graded tapered nanorods in free longitudinal vibration. Bending and buckling of FG nanobeams were examined by Şimşek and Yurctu [29] using an analytical approach. Kiani [30] proposed a mathematical model to explore vibrations and instabilities of moving FG nanobeam by implementing nonlocal Rayleigh beam model. Rahmani and Pedram [31] analyzed the size effect on vibration of nonlocal Timoshenko beam. Finally, Nazemnezhad and Hosseini-Hashemi [32] implemented conventional averaging technique to obtain nonlinear natural frequency of functionally graded beams. However, to the best knowledge of the authors, no study has focused neither on buckling behavior nor on the effect of elastic foundation on vibrational and buckling behavior of nonlocal functionally graded beam.

In the present study, an attempt is made to investigate the free vibration and buckling behavior of functionally graded nanobeams resting on nonlinear elastic foundation. Application of Galerkin technique to the equation of motion of the beam results in a second order nonlinear ordinary equation as the governing equation of the problem. Using He's variational method, a closed form solution is obtained for natural frequency of nonlocal FG beams. Finally, through some numerical examples, the effects of various parameters such as nonlocality, boundary conditions, material inhomogeneity and nonlinearities of the system are investigated.

2. Governing equations

2.1. Material properties

A functionally graded nanobeam with length L, thickness h and width b is depicted in Fig. 1. Different material properties such as elastic modulus, E, mass density, ρ, Poisson's ratio, v, is considered to vary according to power-law form which can be described by

$$P(z) = (P_U - P_L)\left(\frac{z}{h}+\frac{1}{2}\right)^{n_0} + P_L \tag{1}$$

in which P_u and P_L may be any of the above material properties at the upper and lower surfaces of the beam, respectively, and index n_0 indicates the variation profile of material properties across the thickness of the nanobeam. According to this distribution function, for $n_0 = 0$ there is no material inhomogeneity in the beam and it may be considered an isotropic beam with bulk properties of the upper surface.

2.2. Nonlocal effects

In the classic elastic continuum theory, the stress field at a point $\boldsymbol{X}$ only depends on the strain field at the same point. However, according to Eringen's nonlocal elasticity theory, stress field at a point is dependent on strains at all other points of the body. Therefore, the nonlocal stress tensor σ^{nl} at point $\boldsymbol{X}$ is defined by

Table 1
The mode shape functions of a uniform beam for the different boundary conditions [21].

B.C.	Mode shape function
SS	$\sin(\pi x)$
CS	$(\cosh(qx)-\cos(qx)) - \frac{\cosh(q)-\cos(q)}{\sinh(q)-\sin(q)}(\sinh(qx)-\sin(qx));\quad q = 3.9266$
CC	$(\cosh(qx)-\cos(qx)) - \frac{\cosh(q)-\cos(q)}{\sinh(q)-\sin(q)}(\sinh(qx)-\sin(qx));\quad q = 4.7300$

Table 2
The material properties of the constituent material of the FG beam [32].

Material		Young modulus [GPa]	Poisson's ratio	Density [kg/m^3]
Metal	Aluminum	70	0.24	2700
Ceramic	Silicon	210	0.30	2370

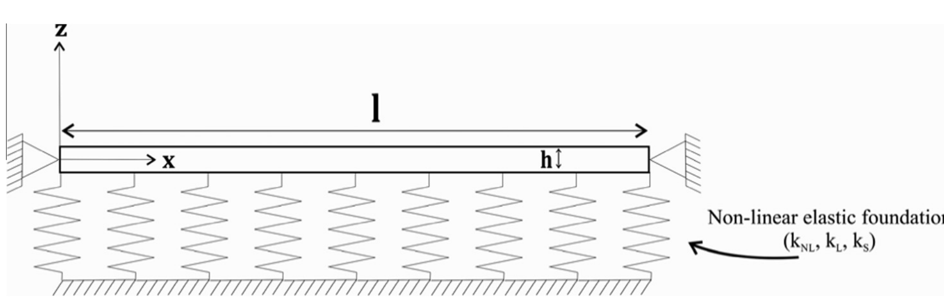

Fig. 1. Nonlocal functionally graded beam resting on nonlinear elastic foundation.