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An exact elasticity model for rib-stiffened plates covered by decoupling acoustic coating layers

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ABSTRACT

This paper presents an exact elasticity model for a fluid-loaded periodically rib-stiffened plate covered by a decoupling acoustic coating layer, which is built upon the plain strain elasticity theory. The acoustical coating layer is assumed to be perfectly bonded to the parallelly rib-stiffened plate, which is impinged by a time-harmonic plane sound wave. The theoretical model begins with Navier–Cauchy equations of motion to describe the vibration behavior of the rib-stiffened plate and the acoustic coating layer, and utilizes the acoustic equation to model the fluid motion. Applying the continuities of displacements and stresses at the interfaces of the plate, the coating layer and the fluid mediums, the vibroacoustic governing equation can be solved to obtain the sound transmission loss (STL) of the structure. The plane strain model is verified by comparing with the thin-plate model, and good agreements have been achieved between the two predictions. Based upon the theoretical model, the influences of the geometrical parameters of the structure, sound incidence angle, the dilatation and shear wave speeds in the acoustic coating layer on the STL of the structure are numerically explored. Results reveal the significant effect of the decoupling acoustic coating layer on the noise reduction of the periodically rib-stiffened structures.

1. Introduction

As an effective passive control approach, the addition of decoupling acoustic coating layers on resonant structures is commonly applied to reduce the vibration and noise radiation by the structures immersed in fluids [1–7]. The purpose of this treatment is to isolate the vibrating structure from the surrounding fluids. Typically, the decoupling acoustic coating layer is made of soft material, which actually introduces impedance mismatching between the structure and the fluid so as to enhance the soundproofing performance of the structure. Previously work mainly concerned with the uniform plates with decoupling coating layers, the present paper aims to develop an elasticity model for the sound transmission characteristic of periodically reinforced plates covered by decoupling acoustic coating layers.

Early researches on the vibroacoustic problems of the decoupling coated structures usually employed the locally reacting model [8–11], which assumed that the decoupling acoustic coating layer behaved as evenly distributed massless springs on the structure. As a progress of theoretical modeling, the elasticity theory is introduced to more accurately account for the dynamics of the decoupling acoustic coating layer and the backing elastic plate. Applying the two-dimensional (2D) plane strain elasticity theory, Chonan and Kugo [1,2] theoretically investigated the transmission characteristics of two- and three-layered infinite plates excited by a plane acoustic wave. Also, in the category of 2D plane strain elasticity theory, Ko [3] developed a theoretical model to evaluate the reduction of structure-borne noise from an infinite plate coated with an air-voided elastomeric baffle. In conjunction with the thin-plate theory, Keltie [4] formulated an analytical model for a compliant elastic coating attached to a submerged thin plate, where the normal and tangential velocity components in the coating were detailed analyzed. Taking into account the attached distributed inhomogeneity, Zhang and Pan [5,6] established a theoretical model to investigate the sound radiation and absorption properties of coated infinite plates.

Previously published researches have mainly focused on the unreinforced plates coated by decoupling acoustic coating layers [1–7]. There are also numerous researches regarding with the reinforced structures [12–22], while which are not covered by decoupling acoustic coating layers. Therefore, the sound transmission performance of periodically rib-stiffened structures covered by decoupling acoustic coating layers is fairly unknown.

To address this deficiency, the present paper formulates a theoretical model for the rib-stiffened plates covered by decoupling acoustic coating layers on the basis of the two-dimensional plain

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strain elasticity theory. The considered structure is impinged by a time-harmonic acoustic wave on the decoupling acoustic coating layer side, and radiates acoustic wave from the rib-stiffeners side. Since the vibrating elastic plate is the main sound source, the sound wave radiated from the rib-stiffeners is neglected. The constitutive equations of elasticity theory are applied to model the dynamics of the elastic plate and the decoupling acoustic coating layer, which have accounted for the dilatational wave and shear wave propagation in the two solid layers. The acoustic equation is employed to describe the sound wave propagation in the two side fluids. The flexural motion and torsional motion of the rib-stiffeners are modeled by the Timoshenko beam theorem and torsional wave equation, respectively. The continuity conditions of displacements and stresses at the fluid–solid and solid–solid interfaces insure the sole solution of the system. Model validation shows an excellent agreement between the present elasticity theoretical model and the thin-plate theoretical model, which also confirms the superiority of the elasticity model for accurate predictions at high frequencies. Specific attentions are then paid to the investigations for the influence of the geometrical parameters, sound incidence angle, the dilatational and shear wave speeds in the decoupling acoustic coating layer on the sound transmission loss of the considered structure.

2. Theoretical formulation

2.1. Governing field equations

As shown in Fig. 1, the considered structure is composed of a periodically rib-stiffened plate covered by a decoupling acoustic coating layer, which is immersed in ideal, inviscid acoustic fluid media. The structure has infinite spatial extent in the x- and z-directions, also with the rib-stiffeners infinite extended in the z-direction. The backing plate and the decoupling acoustic coating layer are both considered as linear, homogeneous and isotropic elastic material, respectively with the thickness of h_b and h_t. The rib-stiffeners on the backing elastic plate are equally spaced at a distance of L in the x direction, with h_r and t being the height and thickness, respectively. The decoupling coating layer and rib-stiffeners are all perfectly bonded on the elastic plate. Considering the system to be a two-dimensional plane strain problem, a time-harmonic plane acoustic wave is assumed to incident on the decoupling coating layer at an angel θ (with respect to the y axis), which only has two wavenumber components k_x and k_y in the x- and y-direction, respectively.

Since the fluid media is considered an ideal and inviscid compressible medium, in which there does not exist shear stresses

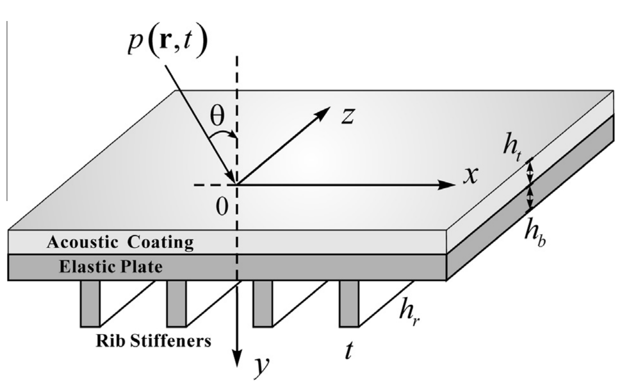

Fig. 1. Schematic illustration of sound transmission through rib-stiffened plate covered by a decoupling acoustic coating layer.

and the propagation of acoustic wave can be conveniently expressed in terms of a scalar velocity potential as:

$$\nabla^2\varphi_i = \frac{1}{c_i^2}\frac{\partial^2\varphi_i}{\partial t^2} \tag{1}$$

$$\mathbf{v}_i = -\nabla\varphi_i, \quad p_i = \rho_i\frac{\partial\varphi_i}{\partial t} \tag{2}$$

where the index $i = 1, 2$ refers to the field variable in the upper and lower fluid media, respectively. ρ_i is the fluid density, $\mathbf{v}_i$ is the fluid particle velocity vector, p_i is the sound pressure and c_i is the sound speed in the fluids. ∇ is the gradient operator and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian. Taking the advantage of the periodicity of the structure, the solutions of the Helmholtz equation in the upper and lower fluid media can be separately written as

$$\varphi_1(x,y,t) = \varphi_{in}e^{-i(k_x x+k_y y-\omega t)} + \sum_{m=-\infty}^{+\infty}\varphi_{r,m}e^{-i(k_{x,m}x-k_{y,m}y-\omega t)} \tag{3}$$

$$\varphi_2(x,y,t) = \sum_{m=-\infty}^{+\infty}\varphi_{t,m}e^{-i(k_{x,m}x+k_{y,m}y-\omega t)} \tag{4}$$

where φ_{in} is the amplitude of the incident sound velocity potential, $\varphi_{r,m}$ and $\varphi_{t,m}$ are the mth amplitude of the reflected and transmitted sound velocity potential, respectively. The wavenumber components k_x and k_y are related to the incident sound angle

$$k_x = k\sin\theta, \quad k_y = k\cos\theta, \quad k = \omega/c_0 \tag{5}$$

here consider that the reflected and transmitted sound wavenumbers have includes the components of structural flexural wavenumbers, thereby one has

$$k_{x,m} = k_x + \frac{2m\pi}{L}, \quad k_{y,m} = \sqrt{k^2 - k_{x,m}^2} \tag{6}$$

Following the relationship of Eq. (2), the normal fluid velocity and sound pressure in the upper and lower fluid media can be obtained, respectively as

$$\begin{aligned} v_{1,y}(x,y,t) &= -\frac{\partial\varphi_1}{\partial y} = ik_y\varphi_{in}e^{-i(k_x x+k_y y-\omega t)} \\ &\quad - \sum_{m=-\infty}^{+\infty} ik_{y,m}\varphi_{r,m}e^{-i(k_{x,m}x-k_{y,m}y-\omega t)} \end{aligned} \tag{7}$$

$$v_{2,y}(x,y,t) = -\frac{\partial\varphi_2}{\partial y} = \sum_{m=-\infty}^{+\infty} ik_{y,m}\varphi_{t,m}e^{-i(k_{x,m}x+k_{y,m}y-\omega t)} \tag{8}$$

$$\begin{aligned} p_1(x,y,t) &= i\omega\rho_1\varphi_1 \\ &= i\omega\rho_1\left(\varphi_{in}e^{-i(k_x x+k_y y-\omega t)} + \sum_{m=-\infty}^{+\infty}\varphi_{r,m}e^{-i(k_{x,m}x-k_{y,m}y-\omega t)}\right) \end{aligned} \tag{9}$$

$$p_2(x,y,t) = i\omega\rho_2\varphi_2 = i\omega\rho_2\sum_{m=-\infty}^{+\infty}\varphi_{t,m}e^{-i(k_{x,m}x+k_{y,m}y-\omega t)} \tag{10}$$

Each layer of the structure is made of linear, homogeneous and isotropic elastic material, for which the Navier–Cauchy equation in the absence of body force is written as

$$\mu_\alpha\nabla^2\mathbf{u}^{(\alpha)} + (\lambda_\alpha + \mu_\alpha)\nabla(\nabla\cdot\mathbf{u}^{(\alpha)}) = \rho_\alpha\frac{\partial^2\mathbf{u}^{(\alpha)}}{\partial t^2} \tag{11}$$

where the index $\alpha = t, b$ refers to the top decoupling coating layer and the bottom elastic plate layer, respectively. $\mathbf{u}^{(\alpha)}$ is the displacement vector, ρ_α is the density, λ_α and μ_α are the lame constants of the solid layers. The stresses can be obtained by the constitutive relations

$$\sigma_{ij}^{(\alpha)} = \lambda_\alpha(\nabla\cdot\mathbf{u}^{(\alpha)})\delta_{ij} + \mu_\alpha\left(u_{i,j}^{(\alpha)} + u_{j,i}^{(\alpha)}\right) \tag{12}$$

The Helmholtz theorem says that the displacement field can be divided into an irrotational part and a rotational part as