Composite Structures 119 (2015) 568–577

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A new family of finite elements for wrinkling analysis of thin films on compliant substrates

Jie Yang [a], Qun Huang [a], Heng Hu [a,*], Gaetano Giunta [b], Salim Belouettar [b], Michel Potier-Ferry [c,d]

[a] School of Civil Engineering, Wuhan University, 8 South Road of East Lake, Wuchang, 430072 Wuhan, PR China
[b] Centre de Recherche Public Henri Tudor, 29, av. John F. Kennedy, L-1855 Luxembourg-Kirchberg, Luxembourg
[c] Laboratoire d'Etude des Microstructures et de Mécanique des Matériaux, LEM3, UMR CNRS 7239, Université de Lorraine, Ile du Saulcy, 57045 Metz Cedex 01, France
[d] Laboratory of Excellence on Design of Alloy Metals for low-mAss Structures (DAMAS), Université de Lorraine, France

ARTICLE INFO

Article history:
Available online 28 September 2014

Keywords:
Finite element
Film/substrate systems
Wrinkling

ABSTRACT

This paper presents a new one-dimensional finite elements' family for the analysis of wrinkling in stiff thin films resting on a thick elastic substrate. Euler–Bernoulli's theory is used for the thin film, whereas the substrate is ideally divided into two parts: 1. a core layer in the neighbourhood of the film where the displacement field presents high gradients (where an higher-order approximation is required) and 2. the remaining part of the substrate or bottom layer where displacements change very slowly. Low-order models allow an accurate yet efficient description of this latter part. Due to its versatility and generality, Carrera's Unified Formulation is used to develop the proposed elements' family. Governing equations' weak form is derived by means of the principle of virtual displacements and discretised in a finite element sense. The asymptotic numerical method is used to solve the resulting non-linear equations' system. Numerical investigations show that the proposed one-dimensional elements are able to capture the instability phenomena in film-substrate systems. In order to validate the proposed finite element models, the critical loads and half-wave numbers predicted by the one-dimensional elements are compared with those obtained via two-dimensional finite element analyses and a very good agreement is found.

1. Introduction

Wrinkling instabilities of different shapes (stripes, labyrinths, or herringbones) are very common in several structural configurations. In particular, they very likely to occur in thin stiff films or coatings resting on compliant substrates and subjected to in-plane mechanical or thermal loads, see Bowden et al. [1]. These systems find several applications in several engineering fields spanning from optics and electronics to aeronautics and space as, for instance, optical and acoustic devices or skin layers in sandwich composites. Furthermore, the interest on soft polymeric substrates is more and more increasing, see Volynskii et al. [2] and Cao and Hutchinson [3]. For these reasons, an accurate yet effective modelling of wrinkling instabilities is a very important and up-to-date research topic.

A brief literature follows. In the framework of sandwich panels design, Allen [4] investigated wrinkles in very stiff thin films when compared to the substrate they resting on. Niu and Talreja [5] determined the critical membrane force and the wrinkles wavelength in sandwich panels on the basis of a linear perturbation analysis. A unified analytical expression for single-sided face, in-phase and out-of-phase wrinkling was presented. Huang et al. [6] proposed a spectral method for modelling wrinkles' evolution where a Winkler foundation (a foundation made of linear springs) was accounted for. The model was extended to the case of a thick elastic foundation in Huang et al. [7] where wavelength and amplitude for various moduli and thicknesses of the substrate in the case of wrinkles (a pattern that is invariant in one direction) were investigated. Audoly and Boudaoud [8–10] studied the straight and herringbones wrinkles as well as the evolution form the former type (due to a moderate load) to the latter one (large buckling) as the external load is increased. Wang et al. [11] investigated both global and local buckling in thin films and membranes resting on elastomeric substrates. An analytical expression for the critical condition separating these two buckling modes was presented and compared with experimental and numerical results. The post-buckling evolution of surface wrinkles was studied by Zang et al. [12] by numerical (finite elements) and analytical models. Several higher-order wrinkling modes were observed. Experiments were also carried

* Corresponding author.
E-mail address: huheng@whu.edu.cn (H. Hu).

http://dx.doi.org/10.1016/j.compstruct.2014.09.040
0263-8223/

out. Stemming form the work by Damil and Potier-Ferry [13], Hu et al. [14] and Xu et al. [15] studied the global and local buckling of long beams resting on a non-linear elastic foundation by means of a multi-scale approach. A Fourier series approximation with slowly varying coefficients was coupled to a refined model by means of the Arlequin method. In such a manner, accurate solutions were obtained at the structural boundaries where the approximated approaches are known to provide poor results.

In this paper, a family of one-dimensional finite elements for the analysis of sinusoidal wrinkling in stiff films resting on thick elastic substrates is presented. A variable through-the-thickness kinematics is implemented by means of Carrera's Unified Formulation (CUF), see Carrera [16], Carrera and Giunta [17–19], Catapano et al. [20] and He et al. [21]. The main aim is to exploit the features of the problem under investigation to propose a family of finite elements that yields accurate results with a number of degrees of freedom as reduced as possible. In particular, the thin film is modelled via Euler–Bernoulli's kinematics. The substrate is ideally divided into two parts where a variable kinematics based upon Taylor's polynomial series expansion is used. The displacement fields expansion order is not a priori fixed but it is a free parameter. The idea of subdividing the substrate aims at effectively and efficiently modelling the substrate mechanical behaviour that plays a very important role in the formation of instability patterns. In a limited neighbourhood of the thin film within the substrate, the displacement field presents a very high through-the-thickness gradient. On the contrary, the remaining part of the substrate is almost unstrained. It is, then, clear that for the former part, called "boundary or core layer", a higher-order kinematics is necessary, whereas a low-order kinematics is sufficient for the substrate "bottom layer". The governing equations are obtained by the virtual work principle and their discrete form is obtained within the framework of the finite element method. The Asymptotic Numerical Method (ANM) is used to solve the non-linear problem, see Damil and Potier-Ferry [22], Cochelin et al. [23,24], Hu et al. [25] and Liu et al. [26]. ANM offers several advantages in terms of computation time and reliability when compared to classical non-linear solution strategies such as Newton–Raphson's and arc-length methods. Analysis investigates the critical wrinkling loads and pattern (in terms of the half-waves number). The effectiveness of different kinematics is studied. Some parametric analyses are carried out to obtain some indications on the appropriate thickness of the core layer. This latter has been related to the wrinkles' wavelength. The proposed elements are assessed towards numerical simulations performed by the commercial finite element software ABAQUS using two-dimensional elements. Very accurate results are obtained and the computational effort is considerably reduced.

2. Model kinematics

A two-dimensional elastic stiff film bound to an elastic soft compliant substrate is considered, see Fig. 1. Skin's and substrate's thickness are addressed by h_f and h_s, whereas h is the total thickness. The substrate is ideally divided into a core and a bottom layer of thickness h_c and h_b. This division allows to describe the rapid variation of the displacement field in the neighbourhood of the thin film by means of high-order kinematic theories, whereas low-order polynomials are used to model the slowly varying kinematics far away from the top membrane. The length and the width of the structure are denoted by L and b. The longitudinal, the through-the-width and the transverse coordinate are x, y and z.

The displacement field is:

$$\mathbf{u}^T(x,z) = \{u(x,z), w(x,z)\} \tag{1}$$

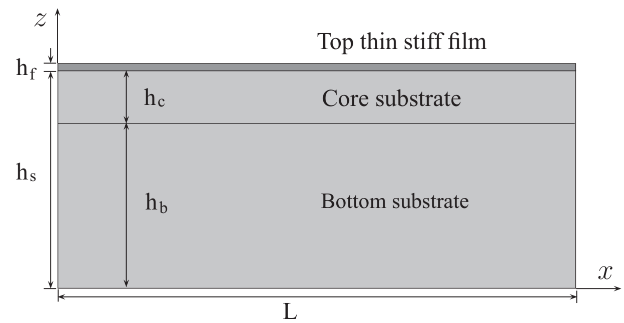

Fig. 1. An elastic thin stiff film on a thick elastic compliant substrate ideally divided into core and bottom layers.

where u and w are the components along x- and z-axis, respectively. T as superscript stands for the transposition operator. According to a one-dimensional modelling approach, the variation along the through-the-thickness direction of the displacement field is a priori assumed. Classically, a polynomial variation of a fixed order n is considered. Within CUF framework, a family of refined beam models can be systematically obtained considering the polynomial approximation order as a free parameter of the formulation, that is, it can assume an arbitrary value. The displacement field in Eq. (1) is approximated as a linear combination of the following monomial terms:

$$\mathbf{u}(x,z) = F_\tau(z)\mathbf{u}_\tau(x) \qquad \tau \in [0, n] \subset \mathbb{N} \tag{2}$$

F_τ represents the through-the-thickness approximating function and, in general, it can be an element of a generic approximation base. Within this work, Mac Laurin's polynomials z^n are adopted as approximation or expansion function. Function $\mathbf{u}_\tau$ accounts for the variation along the beam axis. This latter term depends upon the method used to solve the governing equation. Einstein's compact notation has been used in Eq. (2): a repeated index implicitly implies summation over its variation range:

$$\begin{aligned} u &= u_0 + zu_1 + z^2u_2 + \cdots + z^n u_n \\ w &= w_0 + zw_1 + z^2w_2 + \cdots + z^n w_n \end{aligned} \tag{3}$$

A different kinematic model is independently defined for the thin-film and the substrate core and bottom layers:

$$\mathbf{u}(x,z) : \begin{cases} u^f(x,z) = u_0^f(x) - \left(z - \frac{h_f+2h_s}{2}\right) w_{0,x}^f(x) \\ w^f(x,z) = w_0^f(x) \end{cases} \quad z \in [h_s, h_s + h_f] \tag{4}$$

$$\mathbf{u}(x,z) : \begin{cases} u^c(x,z) = F_\tau u_\tau^c(x) \\ w^c(x,z) = F_\tau w_\tau^c(x) \end{cases} \quad z \in [h_b, h_s] \ \tau \in [0, n_c] \tag{5}$$

$$\mathbf{u}(x,z) : \begin{cases} u^b(x,z) = F_\tau u_\tau^b(x) \\ w^b(x,z) = F_\tau w_\tau^b(x) \end{cases} \quad z \in [0, h_b] \ \tau \in [0, n_b] \tag{6}$$

where superscripts f, c and b stands for film, core and bottom, respectively. Euler–Bernoulli kinematics' is used to model the top film. Through-the-thickness shear and normal deformations are, therefore, disregarded there. This assumption is justified by the thinness of the film. Subscript x preceded by comma stands for differentiation versus the axial coordinate. A CUF variable order kinematics is used for the substrate core and bottom layers where the expansion order is n_c and n_b, respectively.

The continuity of the displacements along the thickness direction is ensured by the following congruency equations:

$$\begin{aligned} u^f(x,h_s) &= u^c(x,h_s) \\ w^f(x,h_s) &= w^c(x,h_s) \\ u^c(x,h_b) &= u^b(x,h_b) \\ w^c(x,h_b) &= w^b(x,h_b) \end{aligned} \quad \forall x \in [0, L] \tag{7}$$